1. Find the real solutions, if any, of
   a. $x^2 + 3x + 9 = 0$

   Discriminant: $b^2 - 4ac = 3 * 3 - 4 * 1 * 9 = -27 < 0$, so NO real solutions

   b. $x^2 - 4x - 2 = 0$

   Discriminant: $b^2 - 4ac = -4 * -4 - 4 * 1 * -2 = 24 > 0$, so 2 real solutions

   Use the **quadratic formula,**

   $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 * 1 * -2}}{2 * 1}$$

   $$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

   $x = 2 + \sqrt{6}, \qquad x = 2 - \sqrt{6}$

   or **complete the square**

   $x^2 - 4x + 4 - 4 - 2 = 0$

   $(x - 2)^2 - 6 = 0$

   $(x - 2)^2 = 6$

   $x - 2 = \pm\sqrt{6}$

   $x = 2 \pm \sqrt{6}$

   $x = 2 + \sqrt{6}, \qquad x = 2 - \sqrt{6}$

2. Calculate:
   a. $3i(-3 + 4i)$

   $3i(-3 + 4i) = -9i + 12i^2$

   $= -12 - 9i$

   b. $\frac{2+3i}{1-i}$

   $\frac{2+3i}{1-i} = \frac{2+3i}{1-i} * \frac{1+i}{1+i} = \frac{2+3i+2i+3i^2}{1-i^2} = \frac{2-3+5i}{1-(-1)} = -\frac{1}{2} + \frac{5}{2}i$

3. Solve $4 - 2x > 1 + x$ for x, express the result as an interval, and draw it on a number line.

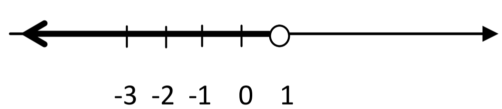
   $4 - 1 > x + 2x$

   $3 > 3x$

   $1 > x$

   $x < 1$

   $x < 1$ or $(-\infty, 1)$



4. Equations of circles:
   a. What are the center and radius of a circle with equation $x^2 + (y+2)^2 = 9$?
      $(x-h)^2+(y-k)^2 = r^2$
      $(x-0)^2 + (y-(-2))^2 = 3^2$

Center is $(0,-2)$ Radius is 3

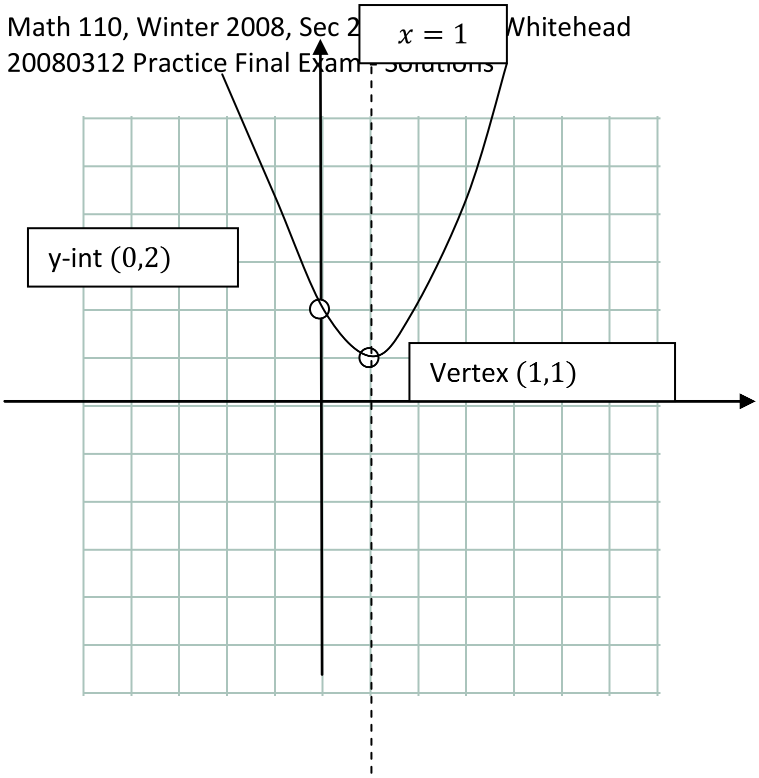
   b. Write an equation of the circle with center at (4,2) and radius 2.
      $(x-h)^2+(y-k)^2 = r^2$
      $(x-4)^2+(y-2)^2 = 2^2$

$(x-4)^2+(y-2)^2 = 4$

5. Put $f(x) = x^2 - 2x + 2$ into the standard form $a(x-h)^2 + k$. Then sketch the graph of $f(x)$ and label
   a. The vertex
   b. The axis of symmetry
   c. The y-intercept
   d. The x-intercepts, if any.

   Standard form - complete the square:

   $f(x) = x^2 - 2x + 1 - 1 + 2$
   $f(x) = (x-1)^2 + 1$
   $a = 1, h = 1, k = 1$
   Vertex: $(1,1)$
   Axis of symmetry: $x = 1$
   Y-intercept: $f(0) = 2 \rightarrow (0,2)$
   X-intercepts: $0 = f(x) = (x-1)^2 + 1$
   $(x-1)^2 = -1$
   No solution so no x-intercepts



6. Which of the following define y as a function of x? (OK just to say “Yes” or “No”)

   a. $\{(-4,4), (-3,4), (-3,1), (-0,0)\}$
   b. $y = |x|$
   c.

X (state)	Y (Senator)
Arizona	McCain
NY	Clinton
Washington	Murray
Arizona	Kyl

a. NO
b. YES
c. NO

7. Find the domain for each of these functions

   a. $f(x) = \sqrt[2]{x+2}$

   $x + 2 \geq 0$

   $x \geq -2$

   b. $g(x) = \frac{x}{x^2 - x}$

   Avoid dividing by 0: $x^2 - x \neq 0$

   $x(x-1) \neq 0$ All $x \neq 0$ $or -1$

c. $h(x) = \ln(x+4)$

Avoid $\ln(0\ or\ negative)$
$x + 4 > 0$

All $x > -4$, or $(-4, \infty)$

8. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$ then evaluate
   a. $f(-1)$

$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$

   b. $g(f(x))$

   $g(f(x)) = \sqrt{f(x)}$

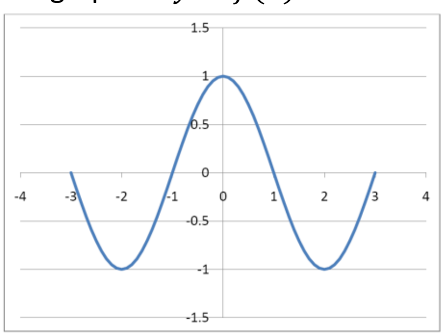
$= \sqrt{x^2 + 1}$

   c. $f(g(x))$

   $f(g(x)) = (\sqrt{x})^2 + 1$

$= x + 1$

9. The graph of $y = f(x)$ is shown below:



   a. Is $f(x)$ even, odd, or neither? EVEN
   b. Is $f\left(\frac{1}{2}\right)$ positive, negative, or zero? Positive
   c. List any intervals on which $f(x)$ is increasing $(-2,0)$and$(2,3)$
   d. Identify any local minima of $f(x)$ $(-2,-1)$ and $(2,-1)$
   e. How often does the line $y = 2$ intersect the graph? 0 times

10. Transformations of graphs

a. What equation is obtained by shifting the graph of $f(x) = \ln(x)$ to the right by 2 units?

Change x by -2: $f(x) = \ln(x-2)$

b. What function has the same graph as $f(x) = x^2$, shifted down by 1 unit?

Change y (outside the function) by -1: $f(x) = x^2 - 1$

11. For each of the following, say if it is a polynomial, and if it is, state the degree

a. $g(x) = \frac{x^4-1}{x^2}$

$$g(x) = \frac{x^4 - 1}{x^2} = x^2 - \frac{1}{x^2}$$

No

b. $f(x) = x^2 + e^x$ No

c. $h(x) = x(x^2+1)$ Yes, degree 3

12. Analyze $f(x) = \frac{x+1}{x-2}$ and provide the following information:

a. Domain
b. Vertical asymptotes if any
c. Horizontal asymptote if any
d. All intercepts
e. Sketch the graph

SOLUTION:

General form: $f(x) = \frac{x+1}{x-2}$

Factored form: $f(x) = \frac{x+1}{x-2}$

Domain: $x \neq 2$

Lowest terms: $f(x) = \frac{x+1}{x-2}$

Vertical asymptotes: $x = 2$

End behavior: Ratio of leading terms $\frac{x}{x} = 1$

When $x \to \infty, f(x) \to 1$

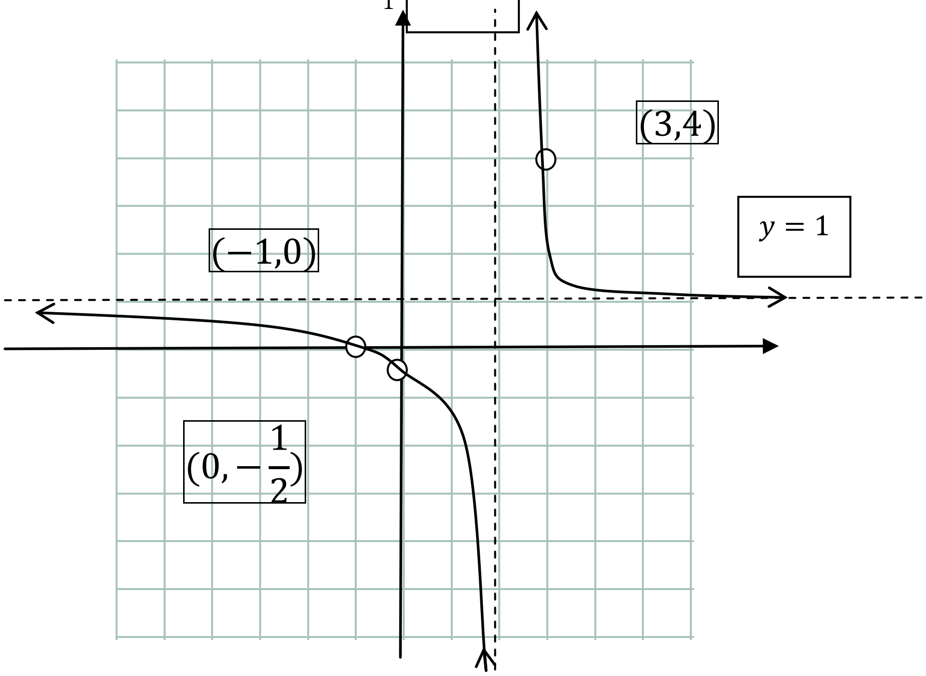
When $x \to -\infty, f(x) \to 1$

Horizontal asymptote: $y = 1$

y-intercept: $f(0) = \frac{1}{-2} = -\frac{1}{2}$ $(0, -\frac{1}{2})$

x-intercepts: $f(x) = 0 \rightarrow x + 1 = 0 \rightarrow x = -1$ $(-1,0)$

Test points: $f(3) = \frac{4}{1} = 4$ $x = 2$ $(3,4)$



13. Say whether each of the following is an *exponential* function of x
    a. $f(x) = \ln(x^2)$ NO
    b. $f(x) = e^2$ No
    c. $f(x) = x^5$ No
    d. $f(x) = 3^x$ Yes

14. Solve $3^{x-2} = 9^x$ for x

$3^{x-2} = 9^x = 3^{2x}$

$x - 2 = 2x$

$x = -2$

15. Logarithms
    a. Write this equation as an equivalent equation involving an exponent: $\log_{10}(y) = 3$

$10^3 = y$

   b. Find $\log_2(16)$

Let $\log_2(16) = x$

$2^x = 16$

$2^4 = 16$

$x = 4$

$\log_2(16) = 4$

16. Logarithms, continued
    a. Write this a single logarithm and simplify if possible
       $3\log_2(u) - \log_2(u^2)$
       $= \log_2(u^3) - \log_2(u^2)$
       $= \log_2(\frac{u^3}{u^2})$
       $= \log_2(u)$
    b. Find $\log_5(712)$ with a calculator by using the LN (natural logarithm) key.

       $\log_5(712) = \frac{\ln(712)}{\ln(5)} = 4.08$

17. You have 100 yards of fencing and will use it to enclose 3 sides of a rectangular lot. The 4th side is the wall of a large building, so it doesn't need fencing.
    a. Express the area *A* of the rectangle as a function of w, the dimension of the rectangle perpendicular to the existing wall.
       $A = lw$
       $100 = 2w + l$
       Solve for l in terms of w: $100 = 2w + l$
       $l = 100 - 2w$
       $A = (100 - 2w)w$
       $A(w) = -2w^2 + 100w$
    b. What is the domain of the function?

       Both l and w must be non-negative: $w \geq 0$ and $l \geq 0$.

       Substitute $l = 100 - 2w$. Then $100 - 2w \geq 0 \rightarrow 2w \leq 100 \rightarrow w \leq 50$

       Putting the information together the domain is $0 \leq w \leq 50$

    c. What is the area if the width is 20 feet?

       $A(20) = -2(20^2) + 100 * 20 = -800 + 2000 = 1600\ sq\ yds$

18. A colony of bacteria grows exponentially. The population is measured to be 2000. Then 2 hours later it is 3,000.
    a. Write an equation for the population as a function of time.
    b. What will the population be at 4 hours?
    c. When will the population be 10,000?

$A(t) = A(0)e^{kt}$

a. Second sentence means $A(0) = 2000$

Third sentence means $\frac{A(2)}{A(0)} = \frac{3}{2} = 1.5$

$1.5 = \frac{A(2)}{A(0)} = e^{k*2}$ since $A(2) = A(0)e^{k*2}$

$\ln(1.5) = \ln\left(\mathrm{e}^{\mathrm{k}*2}\right) = \mathrm{k} * 2$

$\mathrm{k} = \dfrac{\ln(1.5)}{2} = 0.2027$

$A(t) = A(0)e^{kt} = 2000e^{0.2027\,t}$

b. At 4 hours, the population will be $A(4) = 2000e^{0.2027*4} = 4500$

c. Solve $10000 = A(t) = 2000e^{0.2027\,t}$

$5 = e^{0.2027\,t}$

$\ln(5) = 0.2027t$

$t = \frac{\ln(5)}{0.2027} = 7.9387$ hours