Why does $\log_a(u^t) = t\log_a(u)$?

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Let $x = \log_a(u)$ and write that equation as an equivalent one involving exponents:

$a^x = u$

Then $u^t = (a^x)^t = a^{xt}$

And

$\log_a(u^t) = \log_a(a^{xt}) = xt = tx = t\log_a(u)$

Same argument with a specific example:

Why does $\log_2(8^2) = 2\log_2(8)$?

We know $3 = \log_2(8)$ and $2^3 = 8$

Then $8^2 = (2^3)^2 = 2^{3*2}$

And

$\log_2(8^2) = \log_a(2^{3*2}) = 3 * 2 = 2 * 3 = 2\log_2(8)$

Another way to look at it: $\log_2(8)$ answers the question: $2^? = 8$

And $\log_2(8^2)$ answers the question $2^{??} = 8^2$

But $8^2 = \left(2^?\right)^2 = 2^{2*?}$

So the answer to ? ? is the same as 2 * the answer to ?