Sections 5.1, 5.2, 5.4

P. 504 – Example 1

Movie tickets:
You know:

1. Regular price $8, Senior price $6
2. Total number of tickets sold 525
3. Total revenue today $3580

How many of each type of ticket were sold?

$x = \#regular\ tickets,\ y = \#senior\ tickets$

$(1)\ x + y = 525$

$(2)\ 8x + 6y = 3580$

SOLVE BY SUBSTITUTION

SOLVE GRAPHICALLY

$x + y = 525$ means $y = -x + 525$

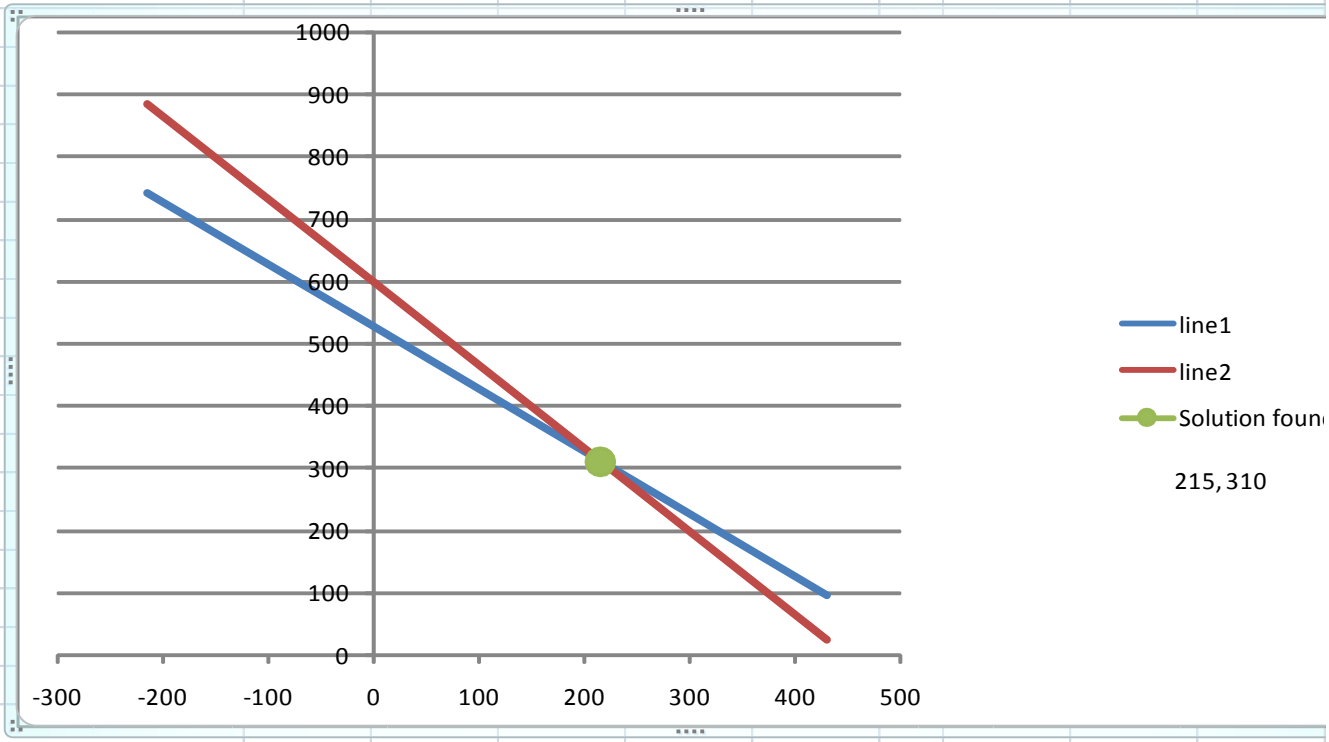
$8x + 6y = 3580$ means $y = -\frac{8}{6}x + \frac{3580}{6}$

These are LINES.

In general, $ax + by = k$ is an equation of a line.

Draw the two lines and see where they intersect

Eq 1	0	1	x	1	y	=	525		-1
Eq 2	0	8	x	6	y	=	3580		-1.33333



Try solving:

$x - y = 3$

$2x + y = 0$

Now try

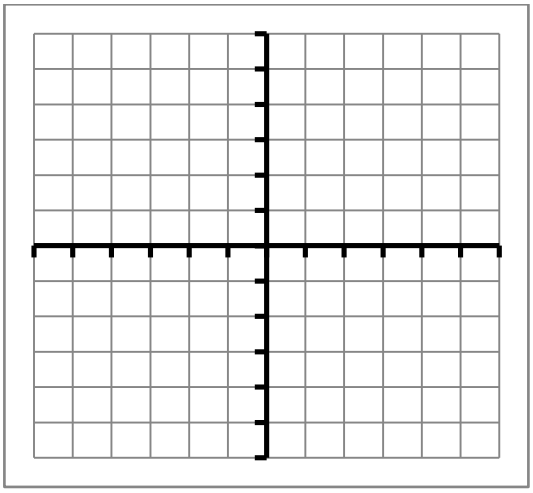
$x + 2y = 5$

$2x + 4y = 6$

Now try

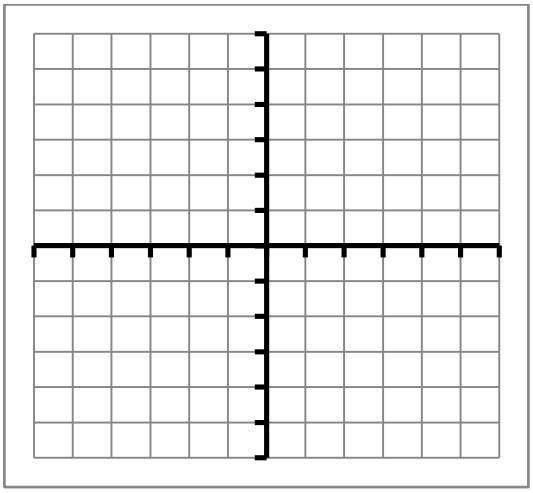
$x + 2y = 5$

$2x + 4y = 6$



$x + 2y = 5$

$2x + 4y = 10$



Practice:

P. 516 #17 Solve

$x + y = 8$

$x - y = 4$

P. 516 #27 Solve

$2x - y = 0$

$3x + 2y = 7$

P. 516 #29 Solve

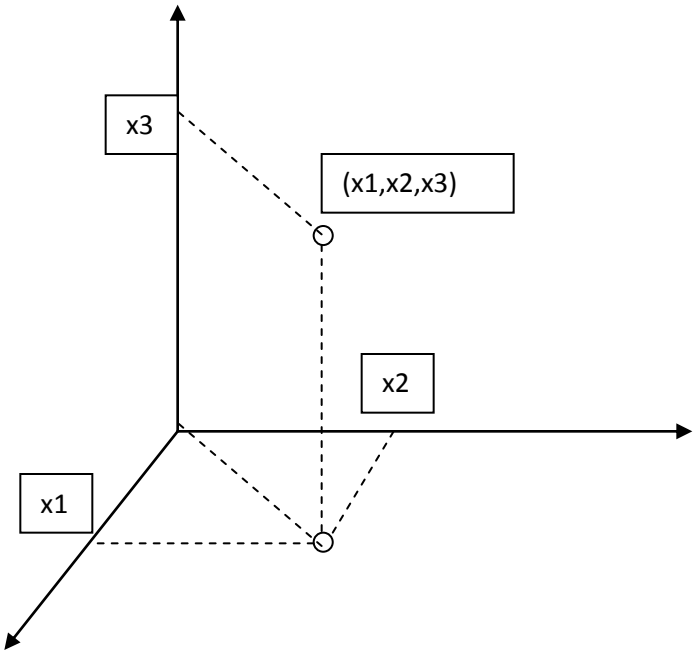
$x + 2y = 4$

$2x + 4y = 8$

Solving 3x3 system by elimination

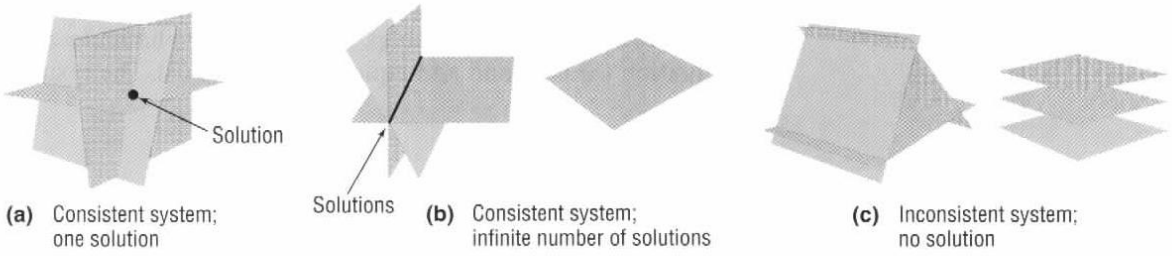
$x_1 + 2x_2 + 2x_3 = 10$
$-7x_1 - 6x_2 - 5x_3 = 20$
$8x_1 + 9x_2 + 10x_3 = 15$

GEOMETRIC SOLUTION



$ax_1 + bx_2 + cx_3 = k$ is an equation of a plane.

Figure 5



(a) Consistent system; one solution

(b) Consistent system; infinite number of solutions

(c) Inconsistent system; no solution

Matrices and “row-echelon form”

$x_1 + 2x_2 + 2x_3 = 10$
$-7x_1 - 6x_2 - 5x_3 = 20$
$8x_1 + 9x_2 + 10x_3 = 15$

“Augmented matrix” for this system

1	2	2	10
-7	-6	-5	20
8	9	10	15

LINEAR EQUATIONS AND MATRIX ALGEBRA

Adding matrices
Multiplying a number (scalar) and a matrix
Multiplying matrices

$x_1 + 2x_2 + 2x_3 = 10$
$-7x_1 - 6x_2 - 5x_3 = 20$
$8x_1 + 9x_2 + 10x_3 = 15$

$$\begin{bmatrix} 1 & 2 & 2 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$\begin{bmatrix} -7 & -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$\begin{bmatrix} 8 & 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

Matrix notation for a 3x3 system
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$
$a_{21}x_1 - a_{22}x_2 - a_{23}x_3 = c_2$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$

$\boldsymbol{Ax = c}$

Write the 3x3 system of equations for:

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & -6 & 0 \\ -4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

Try solving the system by elimination. Then write the solution in matrix form and check it.

More about matrices:

$$\begin{bmatrix} 2 & 2 & 3 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+4+3 \\ -7-12-5 \\ 8+18+10 \end{bmatrix} = \begin{bmatrix} 9 \\ -24 \\ 36 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-0+0 \\ 7-6+5 \\ 8-9+10 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 9 \end{bmatrix}$$

How about

$$\begin{bmatrix} 2 & 2 & 3 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 1 \end{bmatrix} =$$

What matrices can be multiplied? Does the order matter?

If $\boldsymbol{A}$ **and** $\boldsymbol{B}$ are square matrices of the same size, say 3x3, what is the shape of $\boldsymbol{AB}$**?**

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} = \begin{bmatrix}\quad\end{bmatrix} = \begin{bmatrix}\quad\end{bmatrix}$$

$\boldsymbol{I} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ is called the *identity matrix.*

$$\boldsymbol{IA = AI = A}$$

If $\boldsymbol{A}$ ***and*** $\boldsymbol{B}$ are square matrices of the same size and $\boldsymbol{AB = I}$, then

(1) $\boldsymbol{BA = I}$ also
(2) The matrices are *inverses* of each other
(3) We write $\boldsymbol{B = A^{-1}}$

Suppose we have a linear system
$\boldsymbol{Ax = c}$
and we know the inverse $\boldsymbol{A^{-1}}$

Then
$\boldsymbol{A^{-1}Ax = A^{-1}c}$
$\boldsymbol{x = A^{-1}c}$

(Skipping method to calculate inverse…)

$$\begin{bmatrix} 2 & 2 & 3 \\ -7 & -6 & -5 \\ 8 & 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 15 \end{bmatrix}$$

Solve this using calculator

$\boldsymbol{Ax = c}$

$\boldsymbol{x = A^{-1}c}$

Enter A
2nd MATRIX
Arrow right to EDIT
ENTER
3 right arrow 3
(enter the numbers)

Enter C
2nd MATRIX
Arrow right to EDIT
Arrow down to 3: [C]
ENTER
3 right arrow 1
(enter the numbers)

2nd QUIT
CLEAR
2nd MATRIX
1
x^{-1}
*
2nd MATRIX
3
ENTER

How would you solve a larger system?

Matrix inversion:

A										c	check
1	-1	-4	-5	0	-4	2	-3	-4	1	0	-2.2E-15
1	2	2	0	-1	4	2	2	1	0	-1	-1
1	-3	4	-1	-2	3	-3	-2	2	-4	0	5.33E-15
2	-1	1	-3	-1	-5	2	-5	-3	-5	-1	-1
0	2	2	-5	-4	4	-5	-1	-3	-5	-2	-2
2	1	3	-2	-2	-5	3	-1	-2	-2	0	5.33E-15
-1	-5	4	0	-1	-2	3	-1	4	3	-5	-5
1	-5	-1	-5	-1	1	-1	3	1	2	-1	-1
4	1	4	-4	-1	0	1	-4	4	-5	1	1
-2	-3	-4	0	1	-5	3	0	-5	0	0	-7.1E-15

Ainverse										Ainverse*c
0.382097	0.010804	0.59606	-0.51569	-0.28128	0.407449	-0.275	-0.09731	0.029637	0.034358	2.569367
0.026747	-0.04443	-0.08515	-0.15285	0.063241	0.064372	-0.00906	-0.08524	0.097804	0.01762	0.299162
-1.47369	0.418172	-2.05836	3.541248	0.284271	-1.46086	0.529216	0.998522	-1.17229	-1.58648	-9.34485
0.737084	-0.1731	1.200299	-1.73	-0.25025	0.821717	-0.30107	-0.56368	0.432628	0.707807	4.905256
-2.44074	0.637923	-3.48748	5.83312	0.399609	-2.62578	0.743563	1.694414	-1.75668	-2.47574	-14.4392
-0.15767	0.24022	-0.25196	0.597163	0.033347	-0.35421	0.075114	0.13415	-0.22007	-0.25236	-1.63386
0.18981	0.126628	0.219941	-0.43057	-0.03641	0.141335	-0.00625	-0.15951	0.204904	0.312131	0.772409
-0.23569	0.00718	-0.18717	0.134204	0.026704	0.00648	-0.03581	0.173643	-0.01293	0.011718	-0.20231
0.793473	-0.34886	1.14578	-2.22389	-0.12812	0.880933	-0.23809	-0.59972	0.858969	0.989357	5.478115
-0.10816	0.085684	-0.33926	0.634661	0.004182	-0.23424	0.130633	0.165394	-0.35083	-0.46473	-1.8981

Elimination – same idea. Eliminate the variables one at time

A										c
1	-4	-1	0	-5	0	-3	-5	-3	-2	0
0	-1	-2	-3	4	-4	-3	3	3	-4	-5
-5	4	-4	0	0	0	0	-5	-2	-3	0
4	1	-5	-1	-5	1	-4	4	2	-5	-2
1	2	-4	-2	3	-5	4	2	-2	1	2
3	0	-1	1	-3	-2	4	-5	-3	4	0
2	4	-3	0	0	2	1	3	-1	-3	0
1	3	0	-1	-4	2	2	-3	-3	-4	2
-1	-4	-4	3	2	1	-3	4	-1	3	3
1	0	4	-3	-4	-5	3	3	-3	-3	-4

Eliminate x1

1	-4	-1	0	-5	0	-3	-5	-3	-2	0
0	-1	-2	-3	4	-4	-3	3	3	-4	-5
0	-16	-9	0	-25	0	-15	-30	-17	-13	0
0	17	-1	-1	15	1	8	24	14	3	-2
0	6	-3	-2	8	-5	7	7	1	3	2
0	12	2	1	12	-2	13	10	6	10	0
0	12	-1	0	10	2	7	13	5	1	0
0	7	1	-1	1	2	5	2	0	-2	2
0	-8	-5	3	-3	1	-6	-1	-4	1	3
0	4	5	-3	1	-5	6	8	0	-1	-4