

Collaborative Project 3



Oil Spill An oil tanker strikes a sand bar that rips a hole in the hull of the ship. Oil begins leaking out of the tanker, with the spilled oil forming a circle around the tanker. The radius of the circle is increasing at the rate of 2.2 feet per hour.

- (a) Write the area of the circle as a function of the radius r .

$$a(r) = \pi r^2$$

- (b) Write the radius of the circle as a function of time t .

$$r(t) = 2.2t$$

- (c) What is the radius of the circle after 2 hours? What is the radius of the circle after 2.5 hours?

After 2 hours, the radius is $r(2) = 4.4$ feet.

After 2.5 hours, the radius is $r(2.5) = 5.5$ feet.

- (d) Use the result of part (c) to determine the area of the circle after 2 hours and 2.5 hours.

After 2 hours, the area is $a(r(2)) = \pi(r(2))^2 = \pi(4.4)^2 = 19.36\pi$ square feet.

After 2.5 hours, the area is $a(r(2.5)) = \pi(r(2.5))^2 = \pi(5.5)^2 = 30.25\pi$ square feet.

- (e) Determine a function that represents area as a function of time t .

$$A(t) = a(r(t)) = \pi(2.2t)^2 = 4.84\pi t^2$$

- (f) Use the result of part (e) to determine the area of the circle after 2 hours and hours.

After 2 hours, the area is $A(t) = 4.84\pi 2^2 = \pi(4)(4.84) = 19.36\pi$ square feet.

After 2.5 hours, the area is $A(t) = 4.84\pi 2.5^2 = \pi(6.25)(4.84) = 30.25\pi$ square feet.

- (g) Compute the average rate of change of the area of the circle from 2 hours to 2.5 hours.

The average rate of change is

$$\begin{aligned} & \frac{A(2.5) - A(2)}{2.5 - 2} \\ &= \frac{30.25\pi - 19.36\pi}{0.5} \\ &= \frac{(30.25 - 19.36)\pi}{0.5} \\ &= 21.78\pi \text{ square feet/hour} \end{aligned}$$

- (h) Compute the average rate of change of the area of the circle from 3 hours to 3.5 hours.

$$\begin{aligned} & \frac{A(3.5) - A(3)}{3.5 - 3} \\ &= \frac{59.295\pi - 43.56\pi}{0.5} \\ &= \frac{(59.29 - 43.56)\pi}{0.5} = 31.46\pi \text{ square feet/hour} \end{aligned}$$

- (i) Based on the results obtained in parts (g) and (h), what is happening to the average rate of change of the area of the circle as time passes?

The average rate of change increases as time passes.

- (j) If the oil tanker is 150 yards from shore, when will the oil spill first reach the shoreline? (1 yard=3 feet)

The oil spill first reaches the shoreline at the time t that

$$450 = r(t) = 2.2t$$

Solve for t :

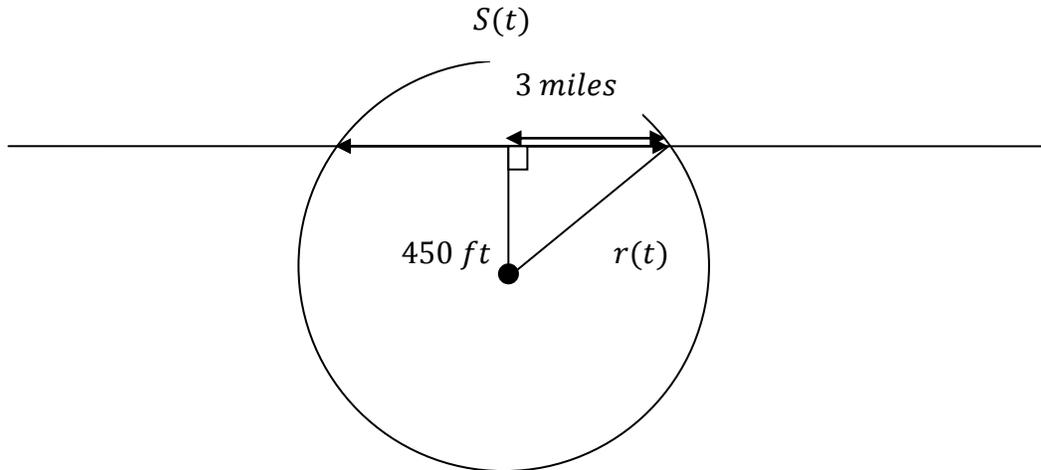
$$450 = 2.2t$$

$$t = \frac{450}{2.2}$$

$$t \approx 204.5455 \text{ hours}$$

(k) How long will it be until 6 miles of shoreline is contaminated with oil? (1 mile=5280 feet).

Let $S(t)$ be the amount of shoreline (in feet) contaminated with oil at time t .



$$3 \text{ miles} = 15840 \text{ feet}$$

Apply the Pythagorean theorem for right triangles:

$$r(t)^2 = 15840^2 + 450^2$$

$$(2.2t)^2 = 15840^2 + 450^2$$

$$4.84t^2 = 15840^2 + 450^2 = 251,108,100$$

$$t^2 = \frac{251,108,100}{4.84} = 51,881,838.84$$

$$t = \sqrt{51,881,838.84}$$

$$\boxed{t \approx 7202.90 \text{ hours}}$$