

SOLUTION

1. What is the domain of  $f(x) = \ln(2x + 1)$ ?

The domain of  $\ln(x)$  is  $\{x: x > 0\}$

So the domain of  $f(x) = \ln(2x + 1)$  is  $\{x: 2x + 1 > 0\}$

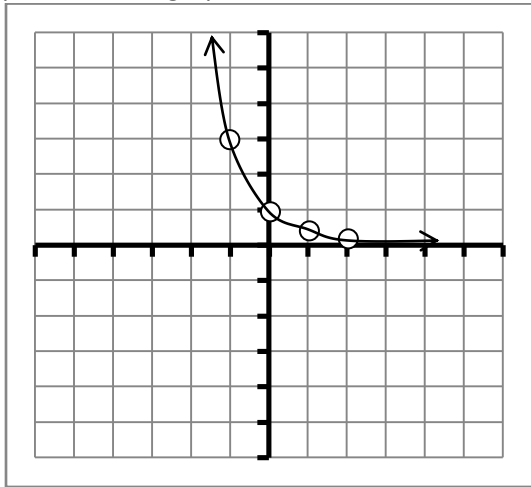
$$2x + 1 > 0$$

$$2x > -1$$

$$x > -\frac{1}{2}$$

or  $\{x: x > -\frac{1}{2}\}$

2. Sketch the graph of  $g(x) = 3^{-x}$ . Make a table of at least 4 values and mark the corresponding points on the graph.



$x$	$g(x)$
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$

3. Solve for  $x$ :

a.  $e^{3x} = 10$

Solution:  $\ln(e^{3x}) = \ln(10)$

$$3x = \ln(10)$$

$$x = \frac{\ln(10)}{3}$$

b.  $\log_2(x) + \log_2(x + 2) = 3$

Solution:

$$\log_2(x(x + 2)) = 3$$

$$2^{\log_2(x(x+2))} = 2^3$$

$$x(x + 2) = 8$$

$$x^2 + 2x = 8$$

$$(x + 1)^2 - 1 = 8$$

$$(x + 1)^2 = 9$$

$$x + 1 = \pm 3$$

$$x = -1 \pm 3$$

$$x = -4 \text{ or } 2$$

Check:  $x = -4$  leads to log of negative number so it is NOT OK

$x = 2$ :  $\log_2(2) + \log_2(4) = 1 + 2 = 3$  OK

Solution:  $x = 2$

SOLUTION

4. If  $\ln(2) = a$  and  $\ln(3) = b$  then answer the following in terms of  $a$  and/or  $b$ .

a.  $\ln(6)$

Solution:  $\ln(6) = \ln(2 * 3) = \ln(2) + \ln(3) = a + b$

b.  $\ln(9)$

Solution:  $\ln(9) = \ln(3^2) = 2 \ln(3) = 2b$

5. Write each of the following as single logarithm.

a.  $3 \log_5(u) - 4 \log_5(v)$

Solution:  $3 \log_5(u) - 4 \log_5(v) = \log_5(u^3) - \log_5(v^4) = \log_5\left(\frac{u^3}{v^4}\right)$

b.  $\log_3(\sqrt{x}) - \log_3(x^3)$

Solution:  $\log_3(\sqrt{x}) - \log_3(x^3)$

$= \log_3(x^{\frac{1}{2}}) - \log_3(x^3)$

$= \log_3\left(\frac{x^{\frac{1}{2}}}{x^3}\right)$

$= \log_3(x^{-\frac{5}{2}})$

$= -\frac{5}{2} \log_3(x)$

Any of these would get full credit

6. If \$500 is invested at 8% compounded quarterly, what is the value of the investment after  $2\frac{1}{2}$  years?

Solution:  $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$A = ? P = \$500 r = 8\% = 0.08 t = 2.5 n = 4$

$A = 500 \left(1 + \frac{0.08}{4}\right)^{2.5 * 4} = 500(1.02)^{10} = \$609.5$

7. Exponential growth and decay:

- a. The element Cilium decays according to the function  $A(t) = A(0)e^{-0.003t}$  where  $A(t)$  is the amount present after  $t$  years. If you begin with 1000 grams of Cilium, how much will be left in 10 years?

Solution:  $A(10) = 1000e^{-0.003 * 10} = 970.4 \text{ grams}$

- b. Referring to part a., when will there be exactly 100 grams of Cilium?

Solution: solve  $100 = A(t) = 1000e^{-0.003t}$

$0.1 = e^{-0.003t}$

$\ln(0.1) = -0.003t$

$t = -\frac{\ln(0.1)}{0.003} = 767.5 \text{ years}$

- c. A colony of bacteria grows exponentially. The colony is measured and weighs 2 grams. Then 2 hours later it weighs 3 grams. Write an equation for colony's weight as a function of time.

Solution:

$$A(t) = A(0)e^{kt}$$

$$A(0) = 2$$

$$A(2) = 3$$

$$A(2) = A(0)e^{k \cdot 2}$$

$$3 = 2e^{k \cdot 2}$$

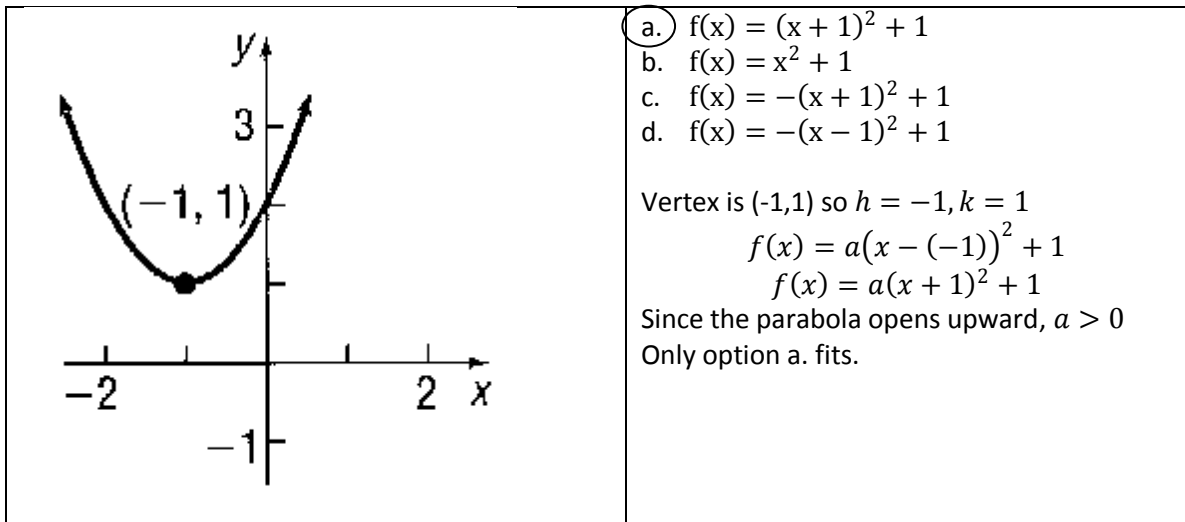
$$1.5 = e^{k \cdot 2}$$

$$\ln(1.5) = k \cdot 2$$

$$k = \frac{\ln(1.5)}{2} = 0.2027$$

$$A(t) = 2e^{0.2027t}$$

8. Which of the given functions could have this graph? (There may be 0, 1, or more than 1 correct answer listed. Circle all that are correct).



9. For the quadratic function  $f(x) = x^2 - 4x$

- a. Put the function in standard form  $f(x) = a(x - h)^2 + k$

$$f(x) = x^2 - 4x = (x - 2)^2 - 4$$

$$a = 1, h = 2, k = -4$$

- b. What is the vertex?

$$(2, -4)$$

- c. What is the axis of symmetry?

$$x = 2$$

- d. Find the x-intercepts and y-intercepts

x-intercepts: solve  $f(x) = 0$

$$x^2 - 4x = 0$$

SOLUTION

$$x(x - 4) = 0$$

$$x = 0, x = 4$$

$$y\text{-intercepts: } y = f(0) = 0$$

e. Sketch the graph

