20080310 Practice test 3

SOLUTION

1. What is the domain of $f(x) = \ln(2x+1)$?

   The domain of $\ln(x)$ is $\{x: x > 0\}$

   So the domain of $f(x) = \ln(2x+1)$ is $\{x: 2x+1 > 0\}$

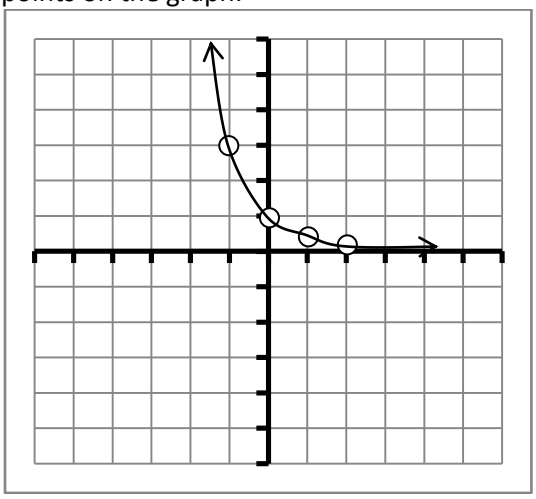
   $2x + 1 > 0$

   $2x > -1$

   $x > -\frac{1}{2}$

   or $\left\{x: x > -\frac{1}{2}\right\}$

2. Sketch the graph of $g(x) = 3^{-x}$. Make a table of at least 4 values and mark the corresponding points on the graph.



x	$g(x)$
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$

3. Solve for x:

   a. $e^{3x} = 10$

   Solution: $\ln(e^{3x}) = \ln(10)$

   $3x = \ln(10)$

   $x = \frac{\ln(10)}{3}$

   b. $\log_2(x) + \log_2(x+2) = 3$

   Solution:

$$\log_2\big(x(x+2)\big) = 3$$
$$2^{\log_2(x(x+2))} = 2^3$$
$$x(x+2) = 8$$
$$x^2 + 2x = 8$$
$$(x+1)^2 - 1 = 8$$
$$(x+1)^2 = 9$$
$$x + 1 = \pm 3$$
$$x = -1 \pm 3$$
$$x = -4 \; or \; 2$$

   Check: $x = -4$ leads to log of negative number so it is NOT OK

   $x = 2: \log_2(2) + \log_2(4) = 1 + 2 = 3 \; OK$

   Solution: $x = 2$

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4. If $\ln(2) = a$ and $\ln(3) = b$ then answer the following in terms of a and/or b.
   a. $\ln(6)$
      Solution: $\ln(6) = \ln(2 * 3) = \ln(2) + \ln(3) = a + b$

   b. $\ln(9)$
      Solution: $\ln(9) = \ln(3^2) = 2\ln(3) = 2b$

5. Write each of the following as single logarithm.
   a. $3\log_5(u) - 4\log_5(v)$
      Solution: $3\log_5(u) - 4\log_5(v) = \log_5(u^3) - \log_5(v^4) = \log_5(\frac{u^3}{v^4})$

   b. $\log_3(\sqrt{x}) - \log_3(x^3)$
      Solution: $\log_3(\sqrt{x}) - \log_3(x^3)$
      $= \log_3(x^{\frac{1}{2}}) - \log_3(x^3)$

      $= \log_3(\frac{x^{\frac{1}{2}}}{x^3})$
      $= \log_3(x^{-\frac{5}{2}})$
      $= -\frac{5}{2}\log_3(x)$

      Any of these would get full credit

6. If \$500 is invested at 8% compounded quarterly, what is the value of the investment after $2\frac{1}{2}$ years?

   Solution: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
   $A = ?\, P = \$500\, r = 8\% = 0.08\, t = 2.5\, n = \ 4$
   $A = 500\left(1 + \frac{0.08}{4}\right)^{2.5*4} = 500(1.02)^{10} = \$609.5$

7. Exponential growth and decay:
   a. The element Cilium decays according to the function $A(t) = A(0)e^{-0.003t}$ where $A(t)$ is the amount present after t years. If you begin with 1000 grams of Cilium, how much will be left in 10 years?

      Solution: $A(10) = 1000e^{-0.003*10} = 970.4\ grams$

   b. Referring to part a., when will there be exactly 100 grams of Cilium?

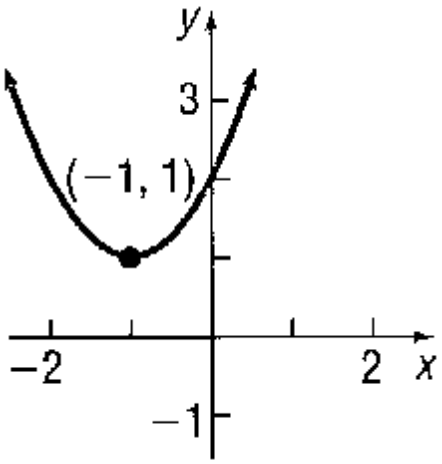
      Solution: solve $100 = A(t) = 1000e^{-0.003t}$
      $0.1 = e^{-0.003t}$
      $\ln(0.1) = -0.003t$
      $t = -\frac{\ln(0.1)}{0.003} = 767.5\ years$

c. A colony of bacteria grows exponentially. The colony is measured and weighs 2 grams. Then 2 hours later it weighs 3 grams. Write an equation for colony's weight as a function of time.

Solution:
$A(t) = A(0)e^{kt}$
$A(0) = 2$
$A(2) = 3$
$A(2) = A(0)e^{k*2}$
$3 = 2e^{k*2}$
$1.5 = e^{k*2}$
$\ln(1.5) = k * 2$
$k = \frac{\ln(1.5)}{2} = 0.2027$

$A(t) = 2e^{0.2027\,t}$

8. Which of the given functions could have this graph? (There may be 0, 1, or more than 1 correct answer listed. Circle all that are correct).



a. $f(x) = (x+1)^2 + 1$
b. $f(x) = x^2 + 1$
c. $f(x) = -(x+1)^2 + 1$
d. $f(x) = -(x-1)^2 + 1$

Vertex is (-1,1) so $h = -1, k = 1$
$$f(x) = a\big(x-(-1)\big)^2 + 1$$
$$f(x) = a(x+1)^2 + 1$$
Since the parabola opens upward, $a > 0$
Only option a. fits.

9. For the quadratic function $f(x) = x^2 - 4x$
   a. Put the function in standard form $f(x) = a(x-h)^2 + k$
   $$f(x) = x^2 - 4x = (x-2)^2 - 2$$
   $$a = 1, h = 2, k = -2$$
   b. What is the vertex?
   $(2, -2)$
   c. What is the axis of symmetry?
   $x = 2$
   d. Find the x-intercepts and y-intercepts
   x-intercepts: solve $f(x) = 0$
   $x^2 - 4x = 0$
   $x((x-4) = 0$
   $x = 0, x = 4$

y-intercepts: $y = f(0) = 0$

e. Sketch the graph