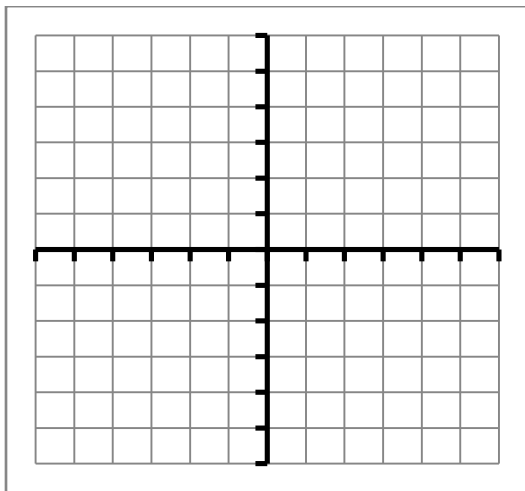


Transformation of $f(x)$	Effect on the graph
$y = f(x - h)$ $y = f(x + h)$	Shift graph to the right by h Shift graph to the left by h
$y = f(ax)$	Shrink the graph horizontally by factor of a
$y = f(-x)$	Reflect the graph about the y axis
$y = bf(x)$	Expand the graph vertically by factor of b
$y = -f(x)$	Reflect the graph about the x axis
$y = f(x) + k$ $y = f(x) - k$	Raise graph by k Lower the graph by k

Example: sketch the graph of $f(x) = (x - 1)^2 + 3$

Starting point:

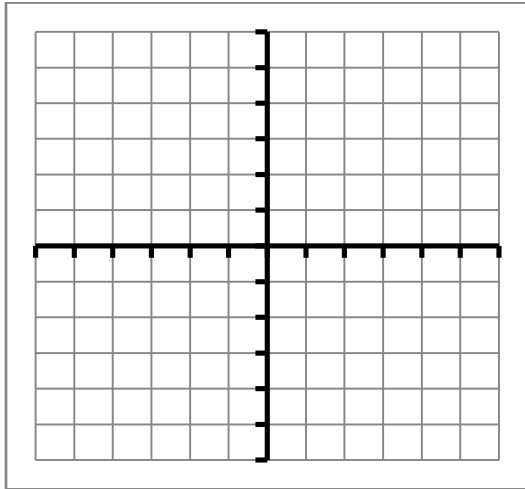
Transformations:



Functions of the form $f(x) = ax^2 + bx + c$ are *quadratic functions*. Their graphs are related to the graph of $f(x) = x^2$. They share the same shape: *parabola*.

The **vertex** of a parabola is the point at its "tip". The **axis of symmetry** is the vertical line at the middle.

What are the vertex and axis of symmetry of $f(x) = x^2$?



Sketch the graph of $f(x) = x^2 - 6x - 1$.

List and draw the vertex, axis of symmetry, and intercepts

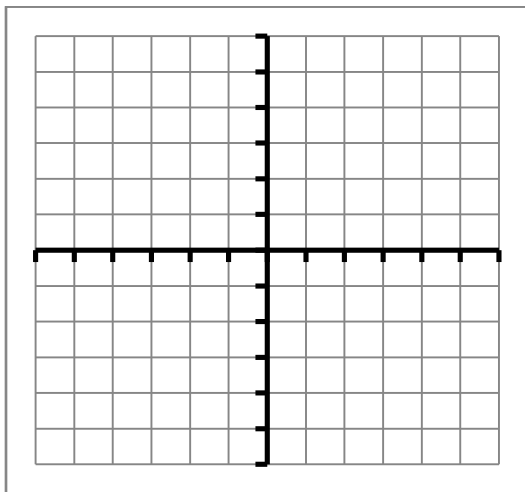
Approach: put this in the form $f(x) = a(x - h)^2 + k$

Complete the square: $f(x) = x^2 - 6x - 1$

Transformations:

Vertex:

Axis of symmetry:



Sketch the graph of $f(x) = -3x^2 + 6x + 2$.

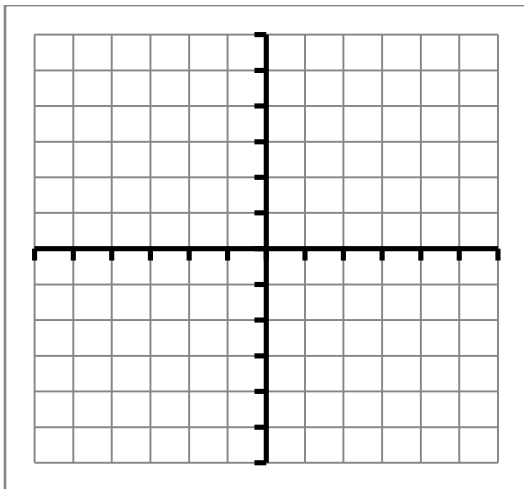
List and draw the vertex, axis of symmetry, and intercepts

Approach: put this in the form $f(x) = a(x - h)^2 + k$

Complete the square: $f(x) = -3x^2 + 6x + 2$

First step: factor out -3 from the x and x^2 terms

Transformation



Sketch the graph of $f(x) = -3x^2 + 6x + 2$.

List and draw the vertex, axis of symmetry, and intercepts

Approach: put this in the form $f(x) = a(x - h)^2 + k$

Complete the square: $f(x) = -3x^2 + 6x + 2$

First step: factor out -3 from the x and x^2 terms

$$f(x) = -3(x^2 - 2x) + 2$$

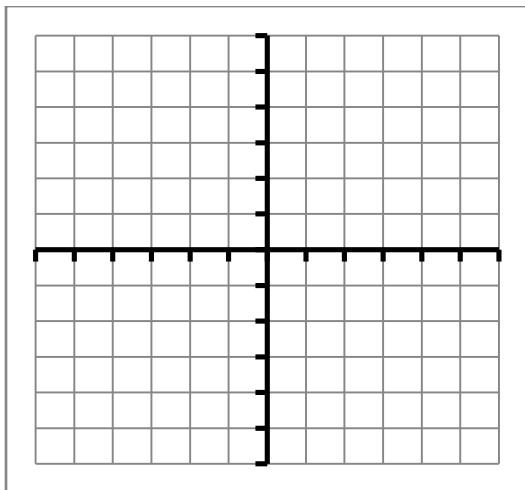
$$= -3(x^2 - 2x + 1 - 1) + 2$$

$$= -3((x - 1)^2 - 1) + 2$$

$$= -3(x - 1)^2 + 3 + 2$$

$$= -3(x - 1)^2 + 5$$

Transformation



Formulas: if $f(x) = ax^2 + bx + c$

Then $f(x) = a(x - h)^2 + k$

$$h = -\frac{b}{2a}$$

$$k = f(h) = f\left(-\frac{b}{2a}\right)$$

The vertex is $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

And the axis of symmetry is the vertical line with equation $x = h$ or $x = -\frac{b}{2a}$

Application: P. 314 #82