Solution to several problems from Section 4.6 – Logarithmic equations
(These were mentioned in the 20080213 class notes)

P. 445, #38: Find the exact value of $log_{1/3}(9)$

If $log_{1/3}(9) = x$

Then $\left(\frac{1}{3}\right)^x = 9$

$$\frac{1^x}{3^x} = 9$$

$$\frac{1}{3^x} = 9$$

$3^{-x} = 9 = 3^2$

Now take $\log_3(\ )$of both sides

$-x = 2$

$x = -2$

P. 445, #40 Find the exact value of $log_5 \sqrt[3]{25}$

$$log_5(\sqrt[3]{25}) = log_5\left(25^{\frac{1}{3}}\right) = \frac{1}{3}log_5(25) = \frac{1}{3}log_5(5^2) = \frac{1}{3} * 2 = \frac{2}{3}$$

P. 445 #52 Find the domain of $f(x) = ln\left(\frac{1}{x-5}\right)$

The domain of $\ln(u)$ is $u > 0$. This is true for ANY log function: the domain of $\log_a(u)$ is $u > 0$.

So the domain of $f(x) = ln\left(\frac{1}{x-5}\right)$Is $\frac{1}{x-5} > 0$ which is the same as $x - 5 > 0$ or $x > 5$.

Solution to several problems from Section 4.6 – Logarithmic equations
(These were mentioned in the 20080213 class notes)

P. 446 #98 Solve $ln(e^{-2x}) = 8$

Remember: $\ln(\quad)$ *means* $\log_e(\ )$

Also remember $\log_a(a^u) = u$

So $ln(e^{-2x}) = \log_e(\ e^{-2x}\ ) = -2x$

So $-2x = 8$

$x = -4$

P. 446 #104 Solve $e^{-2x} = \frac{1}{3}$

Remember $\log_a(a^u) = u$

The key idea: take $\log_e(\ )$ of both sides

$e^{-2x} = \frac{1}{3}$

$\log_e(e^{-2x}\ ) = \log_e(\frac{1}{3})$

$-2x = \log_e(\frac{1}{3})$

$x = -\frac{1}{2}\log_e(\frac{1}{3})$

P. 456 #18 Find $log_3(8)\, log_8(\ 9)$

Solution:

Approach – try to express the logarithms both in the same base. Use the following rule from the cheat sheet. It looks like 3 is the more convenient base to use in common

$\log_b(u) = \frac{\log_a(u)}{\log_a(b)}$

$\mathbf{\log_8(9) = \frac{\log_3(9)}{\log_3(8)}}$

**Substitute this into the original equation:**

$log_3(8)\, log_8(9) = log_3(8)\frac{\mathbf{\log_3(9)}}{\mathbf{\log_3(8)}} = log_3(9) = 2$ **(that's the solution!)**

P. 456 #28 If $ln(2) = a \; and \; ln(3) = b$ Then find $ln(27)$

You are being asked to answer in terms of $a$ and $b$.

As it turns out, $ln(2) = a$ will NOT help us, but $ln(3) = b$ WILL help – because $27 = 3^3$. Use this rule: $\mathbf{\log_a(u^t) = t\log_a(u)}$ **-** then

$ln(27) = \log_e(27) = \log_e(3^3) = 3\log_e(3) = 3b$

P. 456 #44 Express the following using sums and differences of logarithms

$log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right)$

$log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right) = log_5\left(\sqrt[3]{x^2+1}\right) - log_5(x^2-1)$

P. 456 #54 Express the following as a single logarithm

$log_2\left(\frac{1}{x}\right) + log_2\left(\frac{1}{x^2}\right)$

Solution to several problems from Section 4.6 – Logarithmic equations
(These were mentioned in the 20080213 class notes)

$$log_2\left(\frac{1}{x}\right)+log_2\left(\frac{1}{x^2}\right) = log_2\left(\frac{1}{x} * \frac{1}{x^2}\right) = log_2\left(\frac{1}{x^3}\right)$$

P. 460 #10 Solve $3\, log_2(x) = -\, log_2(27)$

Before, we took logs of both sides to resolve exponential functions.
Now do the reverse: apply $2^{()}$ to both sides
$2^{3\, log_2(x)} = 2^{-\, log_2(27)}$

$$\left(2^{log_2(x)}\right)^3 = \left(2^{log_2(27)}\right)^{-1}$$

Why did I think of this re-arrangement? The whole point of the previous step was to get $2^{log_2(something)}$ on each side

So now

$$x^3 = 27^{-1} = \frac{1}{27}$$

Now raise both sides to $\frac{1}{3}$

$$(x^3)^{\frac{1}{3}} = (\frac{1}{27})^{\frac{1}{3}}$$

$$x = \frac{1}{3}$$

P. 460 #14 Solve $log_4(x) + log_4(x-3) = 1$

Two key steps: (1) express the left side as a single log, (2) apply $4^{()}$

(1): $log_4(x) + log_4(x-3) = log_4(x * (x-3)) = 1$

(2): $4^{(log_4(x*(x-3)))} = 4^1 = 4$

$x * (x - 3) = 4$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

Possible solutions are $x = 4$ and $x = -1$. *They need to be checked!*

If $x = 4$ $then\ log_4(x) + log_4(x - 3) = log_4(4) + log_4(4 - 3) = 1 + 0 = 1$ OK

If $x = -1$ $then\ log_4(x) + log_4(x - 3) = log_4(-1) + log_4(-1 - 3)$

But these are logs of negative number so they are NOT DEFINED. So $x = -1$ is NOT a solution.

P. 460 #26 Solve $2^{x+1} = 5^{1-2x}$

One possible approach – find a common base. Since 2 and 5 don't have any common factors, it looks like there is not a convenient common base.

Brute force: take the natural log of both sides (eventually you'll use a calculator).

$\log_e(2^{x+1}) = \log_e(5^{1-2x})$

Apply $\boldsymbol{\log_a(u^t) = t\log_a(u)}$

$(x + 1)\log_e(2) = (1 - 2x)\log_e(5)$

Now just collect terms in $x$ on one side. Remember $\log_e(2)$ and $\log_e(5)$are just numbers!

$\log_e(2) * x + \log_e(2) = \log_e(5) - 2 * \log_e(5) * x$

$(\log_e(2) + 2\ \log_e(5)) * x = \log_e(5) - \log_e(2)$

$x = \frac{\log_e(5) - \log_e(2)}{\log_e(2) + 2\ \log_e(5)} = \frac{\log_e\left(\frac{5}{2}\right)}{\log_e(2) + \log_e(5^2)} = \frac{\log_e(2.5)}{\log_e(50)} \approx 0.234$

Solution to several problems from Section 4.6 – Logarithmic equations
(These were mentioned in the 20080213 class notes)

P. 460 #36 Solve $\log_a(x) + \log_a(x-2) = \log_a(x+4)$

Key steps: (1) Express left side as a single logarithm (2) Apply $a^{(\ \ )}$ to both sides

$\log_a(x) + \log_a(x-2) = \log_a(x*(x-2)) = \log_a(x+4)$

$a^{\log_a(x*(x-2))} = a^{\log_a(x+4)}$

$x(x-2) = x+4$

$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

Now go back and look at P. 460 #14 – a few pages back!