20080220 Practice test 2

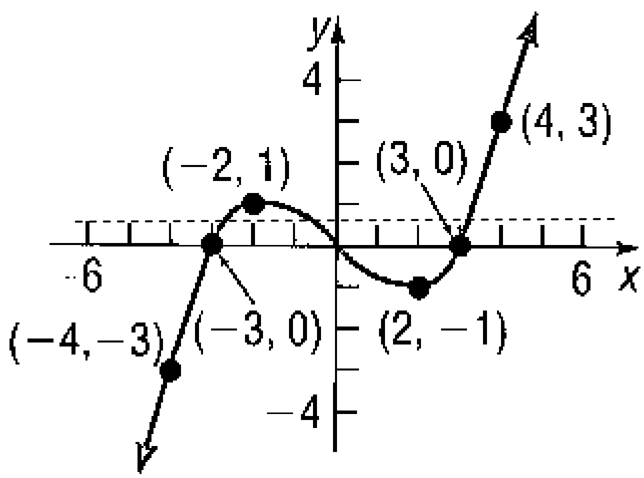
1. The graph of $y = f(x)$ is shown below:
   a. Is $f(x)$ even, odd, or neither? $\boxed{\textbf{ODD}}$

   b. What is $f(-2)$? $\boxed{f(-2) = 1}$

   c. List any intervals on which $f(x)$ is decreasing $\boxed{(-2,2)}$

   d. Identify any local maxima of $f(x)$ $\boxed{x = -2, f(x) = 1}$

   e. How often does the line $y = \frac{1}{2}$ intersect the graph? $\boxed{3}$



2. Let $g(x) = 2x^3$. Evaluate $g(-x)$ and determine whether the function $g(x)$ is even, odd, or neither.

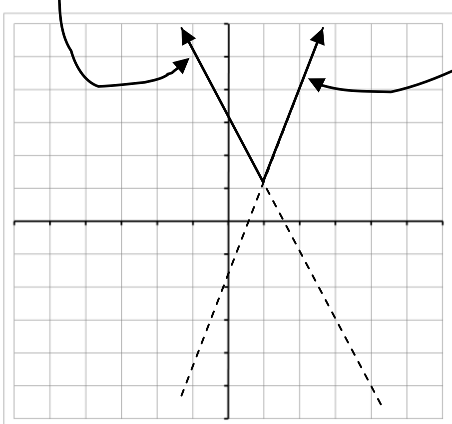
$g(-x) = 2(-x)^3 = 2 * (-1)^3 * x^3 = -2x^3 = -g(x)$

$\boxed{\text{Since } g(-x) = -g(x) \text{ then } g(x) \text{ is ODD}}$

3. Let $f(x) = x^2 - 2$.
   a. Find the average rate of change of $f(x)$ from -2 to 1.
   $$\frac{f(1)-f(-2)}{1-(-2)} = \frac{(1^2-2)-((-2)^2-2)}{3} = \frac{-1-2}{3} = -\frac{3}{3} = -1$$
   b. Find the average rate of change of $f(x)$ from 3 to u.
   $$\frac{f(u)-f(3)}{u-3} = \frac{(u^2-2)-(3^2-2)}{u-3} = \frac{u^2-9}{u-3} = \frac{(u+3)(u-3)}{u-3} = u+3$$

This applies for $u \neq 3$.

4. Let $f(x) = \begin{cases} -2x+3 & x<1 \\ 3x-2 & x \geq 1 \end{cases}$
   a. Find the domain
   All real numbers are included in the definition $x < 1$ and $x \geq 1$, and there are no values that cause problems (division by 0 or $\sqrt{(something < 0)}$) so the $\boxed{\text{domain is all real numbers}}$
   b. Sketch the graph.



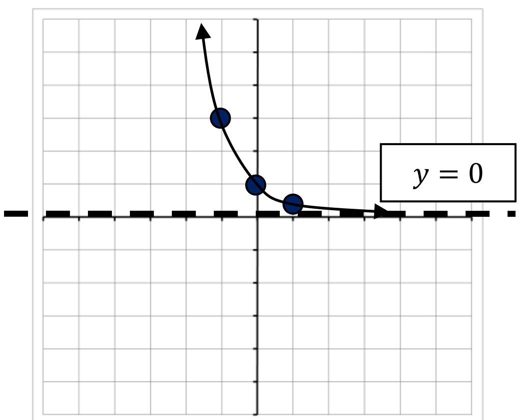
5. Say whether each of the following is an *exponential* function of x
   a. $f(x) = 5^x$ $\boxed{\text{YES}}$

   b. $f(x) = \pi^x$ $\boxed{\text{YES}}$

   c. $f(x) = x^{-3}$ $\boxed{\text{NO}}$

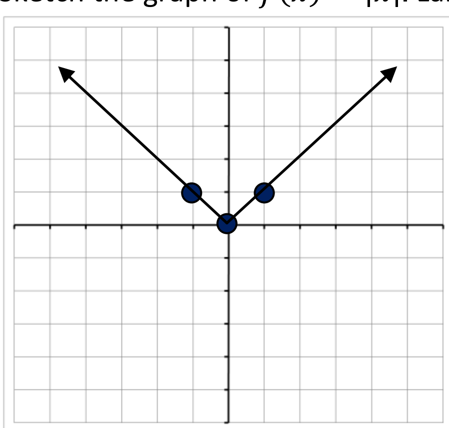
   d. $f(x) = e^{2x}$ $\boxed{\text{YES}}$

6. Sketch the graph of $y = 3^{-x}$. Label 3 points on the graph, and label any asymptotes.



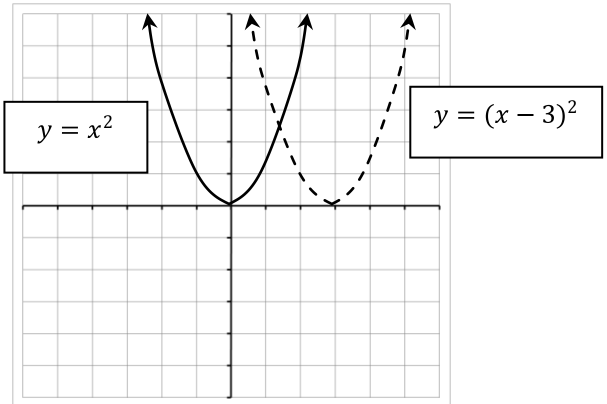
x	y
1	1/3
0	1
−1	3

7. Sketch the graph of $f(x) = |x|$. Label 3 points on the graph.



x	y
1	1
0	0
−1	1

8. Sketch the graph of $y = x^2$. Say how would you use this to sketch the graph of $f(x) = (x-3)^2$ and then go ahead and sketch that too, on the same graph.



Replacing x by $x - 3$ shifts the graph *3 units to the right*

9. If ~~(3,0) (this was my typo on the first version!)~~ $(0,3)$ is on the graph of $y = f(x)$ then which of the following is on the graph of $y = 2f(x)$?
   a. $(0,3)$
   b. $(0,6)$ this one – since $(0,3)$ is on $y = f(x)$ then $f(0) = 3$ so $2f(0) = 2 * 3 = 6$.
   c. $(0,2)$
   d. $(6,0)$

10. If $f(x) = \sqrt{x}$ *and* $g(x) = 2x$ then evaluate:
   a. $f(g(4))$

   $\boxed{f(g(4)) = f(2 * 4) = \sqrt{8} = 2\sqrt{2}}$

   b. $g(f(2))$

   $\boxed{g(f(2)) = g(\sqrt{2}) = 2\sqrt{2}}$ (just coincidence)

   c. $g(g(1))$

   $\boxed{g(g(1)) = g(2 * 1) = 2 * 2 = 4}$

11. Solve $4^{2x} = 64$ for x

A convenient common base is 4.

$64 = 4 * 4 * 4 = 4^3$ so

$4^{2x} = 4^3$

$2x = 3$

$$\boxed{x = \frac{3}{2}}$$

12. The price, *p* (in dollars), of a product and the quantity sold, *x,* are related by the equation

$$x = -20p + 500 \qquad 0 \leq p \leq 25$$

a. Write the revenue R as a function of x.

Definitions:

p	Price per unit of the product	$/unit
x	Quantity sold	#units
R	Total revenue	$

By definition, $R = xp$

Then solve the equation above for p

$x = -20p + 500$

$20p = 500 - x$

$p = \dfrac{500 - x}{20} = 25 - \dfrac{1}{20}x$

Substitute this in $R = xp$

$$\boxed{R(x) = x(25 - \frac{1}{20}x)}$$

b. What is the domain of the function?

This is a "what makes sense?" question.

To make sense, the number units sold must not be negative so $x \geq 0$

And the price must not be negative so $p \geq 0$ which means $25 - \frac{1}{20}x \geq 0$

$25 \geq \frac{1}{20}x$ so $500 \geq x$ or $x \leq 500$

Put these together- the domain is $\boxed{0 \le x \le 500;\ \text{you can also write } [0,500].}$

c. What is the revenue if 20 units are sold?

$$\boxed{R(20) = 20\left(25 - \frac{1}{20} * 20\right) = 20 * 24 = \$480}$$

13. You have 3000 feet of fencing and will use it to enclose a rectangular field. See the illustration below.

a. Express the area *A* of the rectangle as a function of *x*, the length of the rectangle.

Let w be the width and $p = 3000$ is the perimeter.

$A = xw$

$p = 3000 = 2x + 2w$

Solve the 2nd equation for w:

$2w = 3000 - 2x$

$w = 1500 - x$

Substitute: $A = xw = x(1500 - x)$

$$\boxed{A(x) = x(1500 - x)}$$

b. What is the domain of the function?

The length must be at least 0, so $x \ge 0$.

The width must also be at least 0, so $w = 1500 - x \ge 0$ or $x \le 1500$.

So the domain is $0 \le x \le 1500$ or $[0,1500]$

c. What is the area if the length is 1000

$$\boxed{A(1000) = 1000 * (1500 - 1000) = 500{,}000\ sq\ ft}$$