LOGARITHMIC FUNCTIONS

Review of logarithms:

$2^{?} = 8$

$\log_2(8) = \_\_\_\_\_\_$

A logarithm is an *exponent*

- $y = \log_a(x)$ means $a^y = x$
- $\log_a(x)$ is the answer to $a^{?} = x$
- $\log_a x$ is the value of the exponent that must be applied with the base a to produce the outcome x.

Examples:

$\log_2 16 =$

$\log_3 27 =$

$\log_{10} 100 =$

Examples: P. 444, 9-20

In Problems 9–20, change each exponential expression to an equivalent expression involving a logarithm.

9. $9 = 3^2$
10. $16 = 4^2$
11. $a^2 = 1.6$
12. $a^3 = 2.1$
13. $1.1^2 = M$
14. $2.2^3 = N$
15. $2^x = 7.2$
16. $3^x = 4.6$
17. $x^{\sqrt{2}} = \pi$
18. $x^\pi = e$
19. $e^x = 8$
20. $e^{2.2} = M$

An *exponential function* is $f(x) = a^x$ for $a > 0$.

A corresponding *logarithmic function* is $g(x) = \log_a(x)$, and it means $a^y = x$.

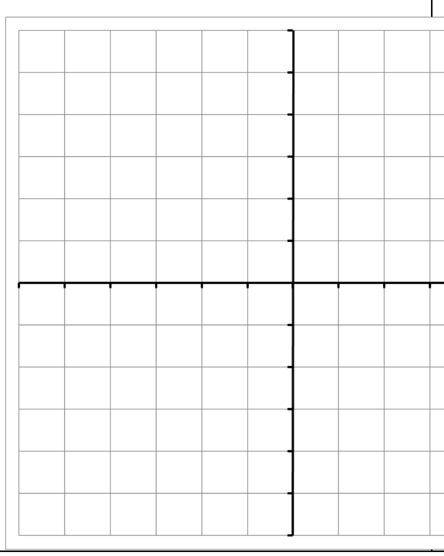
Terminology: $\log_e(x)$ is often written $\ln(x)$, called the *natural log.*

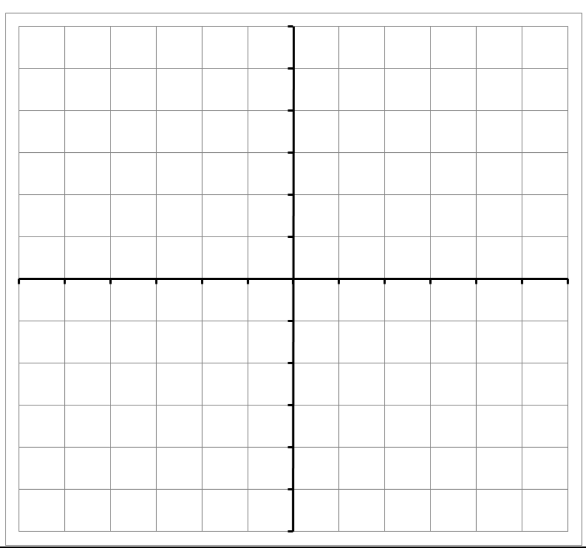
$\log(x)$ - without a base specified – **usually means $\log_{10}(x)$**

Example: sketch the graph of $f(x) = 2^x$ and then sketch the graph of $g(x) = \log_2(x)$. What are the domain and range of each?

x	$y = f(x) = 2^x$
-2	
-1	
0	
1	
2	
3	

x	$y = g(x) = \log_2(x)$ $x =$ ________
	-2
	-1
	0
	1
	2
	3





Rules for exponential functions	**Rules for logarithmic functions**
$a^{s+t} = a^s a^t$	$\log_a(uv) = \log_a(u) + \log_a(v)$
$a^{-x} = \frac{1}{a^x}$	$\log_a\left(\frac{1}{u}\right) = -\log_a u$
$a^0 = 1$	$\log_a(1) = 0$
$a^1 = a$	$\log_a(a) = 1$
$a^{s-t} = \frac{a^s}{a^t}$	$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$
$(a^s)^t = a^{st}$	$\log_a(u^t) = t\log_a(u)$
$(a^s)^t = a^{st}$	$\log_b(u) = \frac{\log_a(u)}{\log_a(b)}$ $\log_a(u) = \log_a(b) \cdot \log_b(u)$

$a^{log_a(x)} = x$	$log_a(a^x) = x$
Range of $f(x) = a^x$ is $(0, \infty)$	Domain of $g(x) = \log_a(x)$ is $(0, \infty)$
Domain of $f(x) = a^x$ is $(-\infty, \infty)$	Range of $g(x) = \log_a(x)$ is $(-\infty, \infty)$
$f(x) = a^x$ has horizontal asymptote $y = 0$	$g(x) = \log_a(x)$ has vertical asymptote $y = 0$
$f(x) = a^x$ has no vertical asymptote	$g(x) = \log_a(x)$ has no horizontal asymptote

How to calculate logs to other bases when only $\mathbf{log}_{\mathrm{e}}(\mathrm{x})$ or $\mathbf{log}_{10}(\mathrm{x})$ is available on your calculator

Use this formula for changing the *base:* $\mathbf{log}_{\boldsymbol{b}}(\boldsymbol{u}) = \frac{\mathbf{log}_{\boldsymbol{a}}(\boldsymbol{u})}{\mathbf{log}_{\boldsymbol{a}}(\boldsymbol{b})}$

For example, $log_3(8)$ with you calculator when it has, say, $ln(x)$ but not $log_3(x)$.

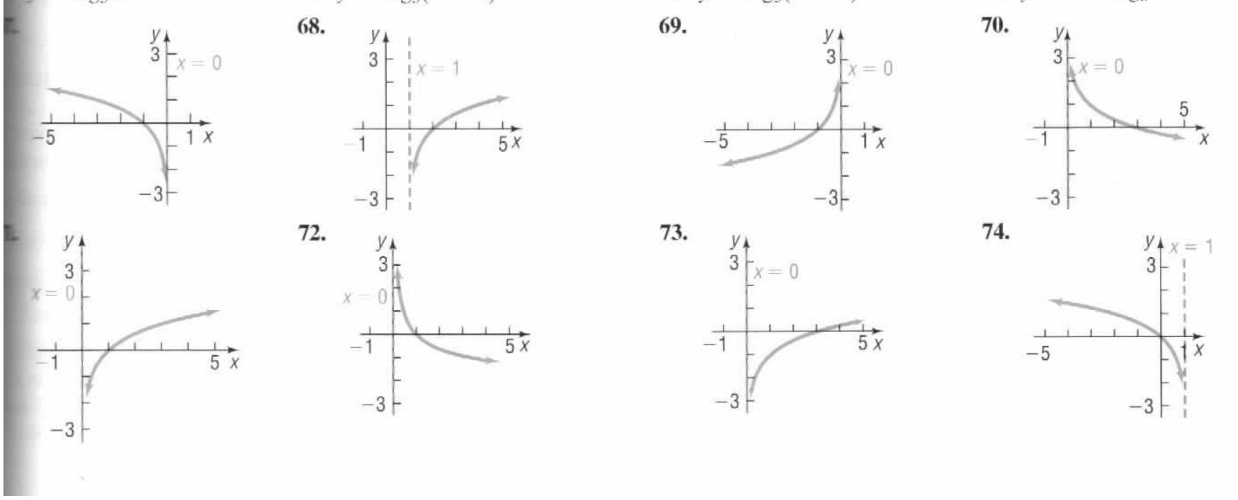
$$\log_3(\mathrm{x}) = \frac{\log_{\mathrm{e}}(\mathrm{x})}{\log_{\mathrm{e}}(3)} = \frac{\ln(\mathrm{x})}{\ln(3)}$$

So $log_3(8) = \frac{\ln(8)}{\ln(3)} = \frac{2.079441542}{1.098612289} = 1.892789261$

P. 445, #38: Find the exact value of $log_{1/3}(9)$

P. 445, #40 Find the exact value of $log_5 \sqrt[3]{25}$

P. 445 #52 Find the domain of $f(x) = ln\left(\frac{1}{x-5}\right)$


Problems 67–74, the graph of a logarithmic function is given. Match each graph to one of the following functions:

$y = \log_3 x$ | *B.* $y = \log_3(-x)$ | *C.* $y = -\log_3 x$ | *D.* $y = -\log_3(-x)$

$y = \log_3 x - 1$ | *F.* $y = \log_3(x-1)$ | *G.* $y = \log_3(1-x)$ | *H.* $y = 1 - \log_3 x$

68. **69.** **70.** **72.** **73.** **74.**

P. 446 #98 Solve $ln(e^{-2x}) = 8$

P. 446 #104 Solve $e^{-2x} = \frac{1}{3}$

P. 456 #18 Find $log_3(8)\, log_8(9)$

Solution:

Approach – try to express the logarithms both in the same base. Use the following rule from the cheat sheet, and it looks like 3 is the more convenient base to use in common

$$\log_b(u) = \frac{\log_a(u)}{\log_a(b)}$$

$$\mathbf{\log_8(9) = \frac{\log_3(9)}{\log_3(8)}}$$

Substitute this into the original equation:

$log_3(8)\, log_8(9) = log_3(8)\frac{\mathbf{\log_3(9)}}{\mathbf{\log_3(8)}} = log_3(9) = 2$ **(that's the solution!)**

P. 456 #28 If $ln(2) = a \text{ and } ln(3) = b$ Then find $ln(27)$

P. 456 #44 Express the following using sums and differences of logarithms

$$log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right)$$

P. 456 #54 Express the following as a single logarithm

$$log_2\left(\frac{1}{x}\right)+log_2\left(\frac{1}{x^2}\right)$$

P. 460 #10 Solve $3\, log_2(x) = -\, log_2(27)$

P. 460 #14 Solve $log_4(x) + log_4(x-3) = 1$

P. 460 #26 Solve $2^{x+1} = 5^{1-2x}$

P. 460 #36 Solve $\log_a(x) + \log_a(x-2) = \log_a(x+4)$

INVERSE FUNCTIONS

Example:

$y = f(x) = 3x - 3$

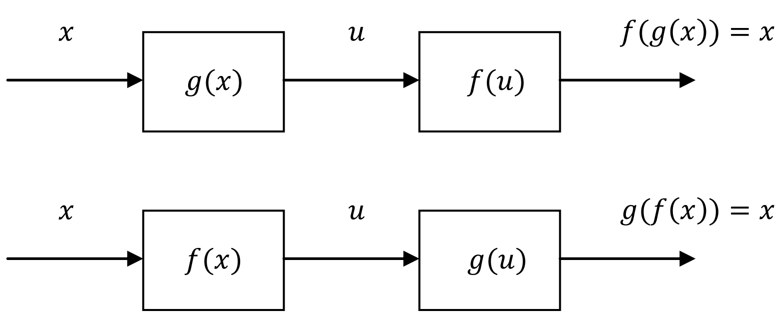
Solve this for x

$x = \frac{1}{3}y + 1$

This defines a function of y

$x = g(y) = \frac{1}{3}y + 1$

Now what is $g\big(f(x)\big)$? How about $g\big(f(x)\big)$?



If $g\big(f(x)\big) = x$ then g is the *inverse* of *f*, $g(x) = f^{-1}(x)$

And in that case $f\big(g(x)\big) = x$ so $f(x) = g^{-1}(x)$

Example:

$y = f(x) = x^2$

Can you find an inverse function?

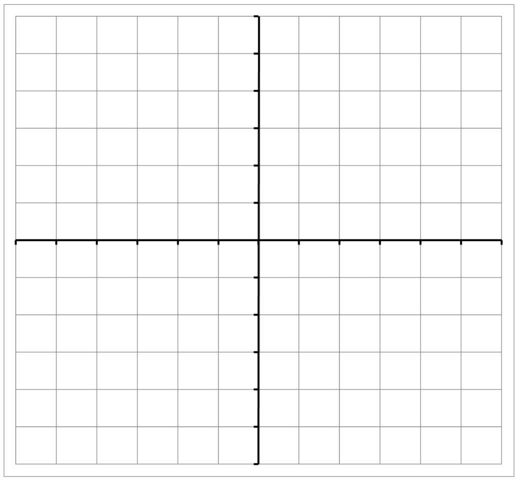
One-to-one functions: not only does each input produce exactly one output, but ALSO, two different inputs always produce to DIFFERENT outputs.

If $f(x)$ is one-to-one, then if $a \neq b$ then $f(a) \neq f(b)$

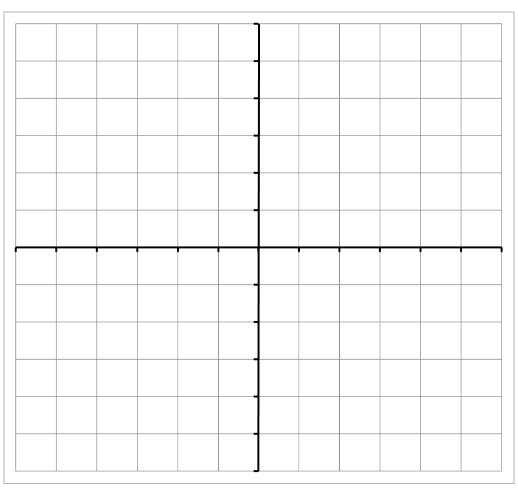
Only one-to-one functions have inverses – and ALL one-to-one functions have inverses.

Find the inverse functions. What is the domain and range of the function? Sketch the function.

52. $f(x) = x^3 + 1$



58. $f(x) = \dfrac{4}{x + 2}$



66. $f(x) = \dfrac{2x - 3}{x + 4}$