COMPOSITE FUNCTIONS

Example:

If $f(x) = x^2 - 2x + 3$, what is $f(x+1)$?

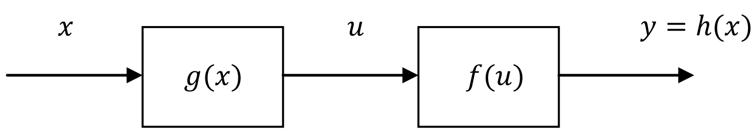
Pattern: $f(\quad) = (\quad)^2 - 2(\quad) + 3$

$f(\,x+1) = (x+1)^2 - 2(x+1) + 3$

or

$f(x) = x^2 - 2x + 3,\ g(x) = x + 1,\ \boldsymbol{h(x) = f(\,g(x)\,)}$

Input-output illustration: for $h(x) = f(\,g(x)\,)$



Evaluating $h(x)$

x	$g(x)$	$f(g(x))$
0	1	2
1	2	3
2		
3		
-1		
-2		
-3		

Expressing $h(x)$ explicitly as a function of x and simplifying:

$h(x) = f(\,x+1) = (x+1)^2 - 2(x+1) + 3$

$= x^2 + 2x + 1 - 2x - 2 + 3$

$= x^2 + 2x - 2x + 1 - 2 + 3$

$h(x) = x^2 + 2$

It's important do know how to do BOTH.

THE ORDER MATTERS!

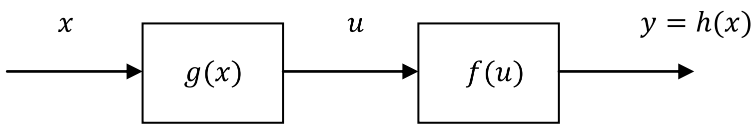
$f(x) = x^2 - 2x + 3,\ g(x) = x + 1,\ H(x) = g(\ f(x)\ )$

$H(x) = g(\ x^2 - 2x + 3) = (x^2 - 2x + 3) + 1$

$H(x) = x^2 - 2x + 4 = (x - 2)^2$

x	$f(x)$	$g(f(x))$

Here (again) is the input-output illustration for $h(x) = f(\ g(x)\ )$



What would the illustration be for $H(x) = g(\ f(x)\ )$?

Composite functions defined by listing – example.

$f(x)$

Income	Tax rate
0-29,999	10%
30,000-59,999	15%
60,000-119,999	25%
120,000 or above	40%

$g(x)$

Person	Income
Smith	24,500
Addai	132,000
Black	60,000
Robbins	1,100,000

$h(x) = f(g(x))$

Person	Tax rate
Smith	
Addai	
Black	
Robbins	

Does $H(x) = g\big(f(x)\big)$ make any sense?

NOTATION: $h = f{\circ}g$ is the same thing as $h(x) = f(\ g(x)\ )$

If $H = g{\circ}f$, then $H(x) =$ ________

Examples

P. 402 #10

a. $g\big(f(1)\big)$

c. $f(g(0))$

P. 402 #12

$f(x) = 3x + 2$ and $g(x) = 2x^2 - 1$

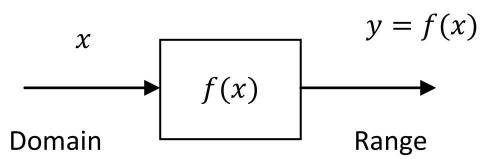
a. $(f^\circ g)(4)$
b. $(g^\circ f)(2)$
c. $(f^\circ f)(1)$

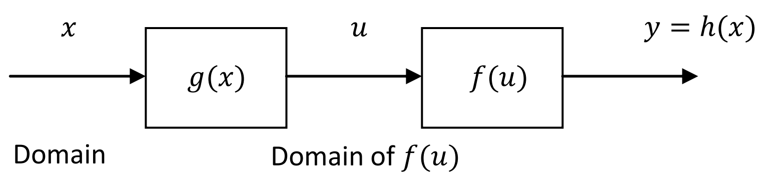
DOMAIN OF COMPOSITE FUNCTION:

Review : domain of a simple function

Domain is the set of possible *inputs* or *values of x.*

- The formula (equation) is defined – no division by 0, no square root of a negative
- Makes physical sense
- Is within any limits you are given explicitly





To find the domain of $h(x) = f\big(g(x)\big)$, work *backwards*

1. Find the domain of $f(u)$
2. That provides conditions on the output of $g(x)$
3. Solve for x
4. Apply any additional constraints on the domain of $g(x)$ itself.

example:

If $f(x) = \sqrt{x}, g(x) = x + 3$

a) Write $h(x) = f\big(g(x)\big)$ explicitly as a function of x.
b) Find the domain of $h(x)$.

example: If $f(x) = \frac{1}{x+3}$, $g(x) = -\frac{2}{x}$

a) Write $h = f^\circ g$ explicitly as a function of x.
b) Find the domain of $h(x)$.

EXPONENTIAL FUNCTIONS

Review exponents

Terminology: $base^{exponent}$

Examples:

3^2 x^4 2^x

Operations with exponents:

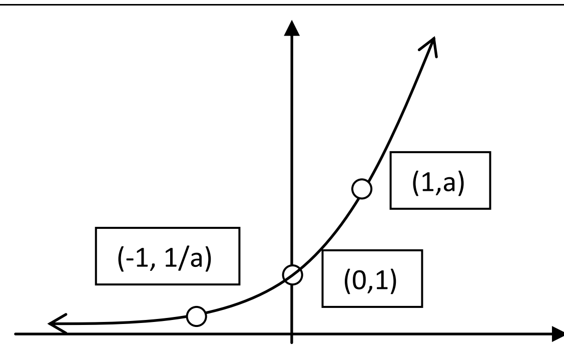
- $3^2 3^3 =$
- $2^{-3} =$
- $64^{1/3} =$
- $3^0 =$
- $1^{200} =$

With these rules, we can calculate, for example, 4^x for any rational number x. (A rational number is one that can be expressed as a ratio of integers $x = {}^m/_n$.)

With other techniques it's also possible to calculate such functions for *irrational* numbers such as $\sqrt{2}$ and π.

Exponential functions: if a is a positive real number, then $f(x) = a^x$ is an exponential function.

- **The domain of f is all the real numbers $(-\infty, \infty)$**
- $\boldsymbol{f(u+v) = f(u)f(v)}$
- $\boldsymbol{f(x+1) = af(x)}$ **or** $\boldsymbol{\frac{f(x+1)}{f(x)} = a}$
- $\boldsymbol{f(-x) = \frac{1}{f(x)}}$
- $\boldsymbol{f(0) = 1, f(1) =}$ ____, **and** $\boldsymbol{f(-1) =}$ ____
- **If $a > 1$ then as $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to 0$**
- **What is the range of $f(x)$?**

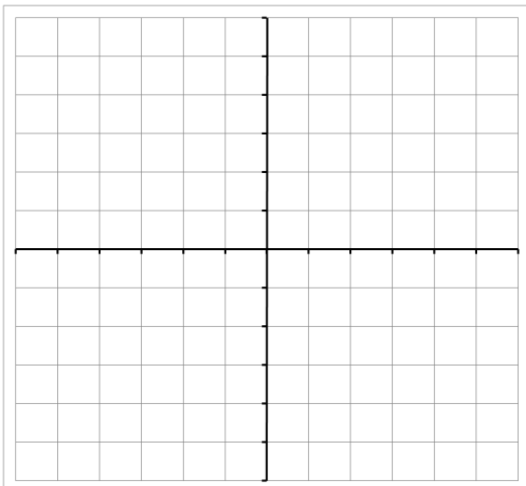


Is $f(x) = x^3$ an exponential function?

If $f(x) = 3^x$ then

$$\frac{f(x+1)}{f(x)} =$$

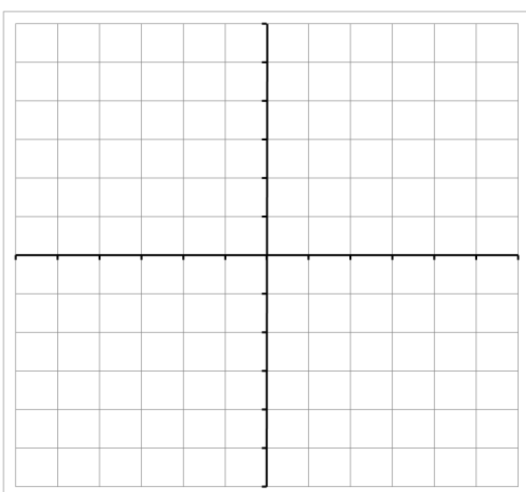
Graph $f(x) = 3^x$

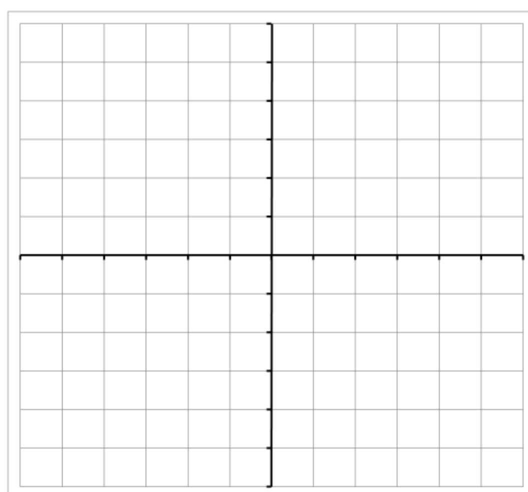


Example: p. 431 #s 21-28

Example: p. 432 #40

Graph $f(x) = -3^x + 1$





The number e and THE exponential function e^x

Formally: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

This means: as $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e$

See p. 427 of the textbook.

$e \approx 2.71828$ **and is an *irrational* number.**

e^x**is called "the exponential function" and is also written** $exp(x)$.

Why do we care about $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$**?**

Example: continuously compounded interest.

Suppose The Money Shrub loans you $1 and charges you 100% annual interest.

How much do you owe after 1 year? Simple interest: $Amount = Principal * (1 + rate * time)$ **or** $A = P(1 + rt)$

$A =$

The Money Shrub has a policy of compounding interest monthly. How much do you owe at the end of one year? Compound interest: $A(1\ month) = P\left(1 + r\frac{1}{12}\right) =$

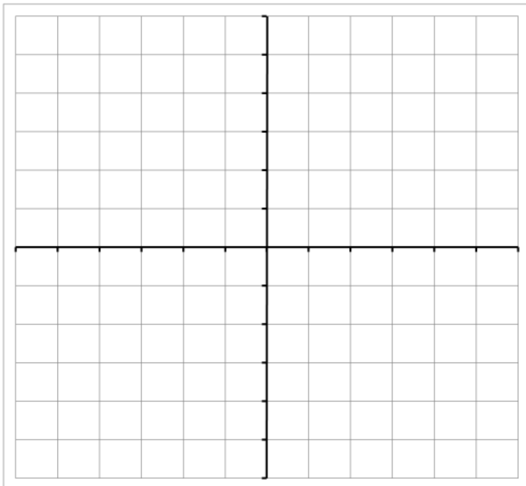
$A(2\ months) =$

…

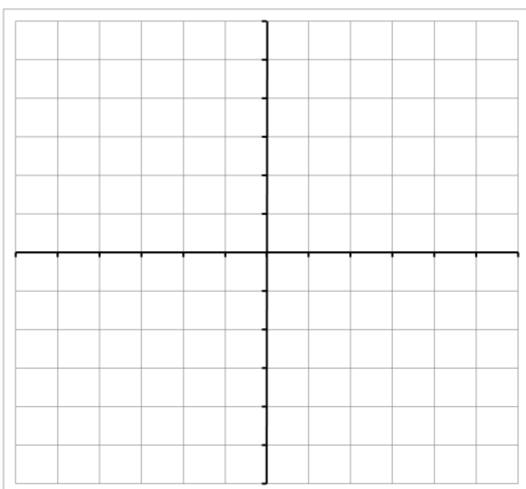
$A(12\ months) =$

Continuously compounded interest: divide the year up into n time periods

Graph $f(x) = e^x$



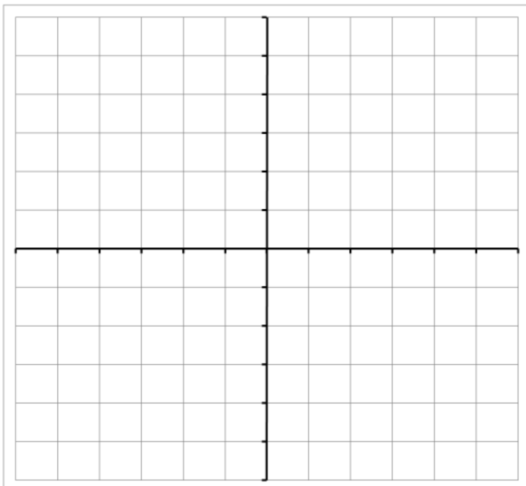
Graph $f(x) = e^{-x}$

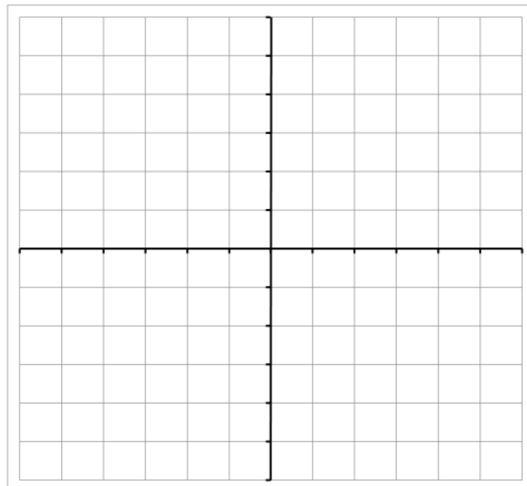


Example: p. 432, #50

Graph $f(x) = 9 - 3e^{-x}$

Transformations:





Solving equations involving exponential functions

KEY STEP: select a single *common base* and express all the terms in that common base.

GOAL: get to $a^{f(x)} = a^{g(x)}$, then you can just solve $f(x) = g(x)$

Solve $5^{1-2x} = \frac{1}{25}$

Common base?

Applications:

On the average, the price of a share of Giigle stock appreciates 9% per year.

a. **If Giigle trades at $150.00 today, what do you expect the price to be in 3 years?**

   GOAL: express the price as a function of time $G(t)$

   KEY STEP: what does "appreciates 9% per year" mean?

b. **How long would you expect to wait until Giigle trades at $300.00?**