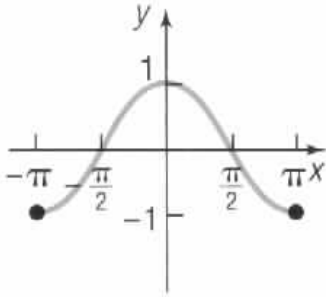


Student name: SOLUTION

Questions 1-12 are worth a total of 100 points. Questions 13-16 are for extra credit, worth 5 points each.

1. List the intercepts of this graph



Y intercept:  $(0, 1)$   
X intercepts:  $(\frac{\pi}{2}, 0)$  and  $(-\frac{\pi}{2}, 0)$

2. Solve for  $x$  by completing the square. Show your work.

$$x^2 - 6x = 27$$

The square will be  $(x - 3)^2$

$$x^2 - 6x + 9 = 27 + 9$$

$$(x - 3)^2 = 36$$

$$x - 3 = \pm\sqrt{36} = \pm 6$$

$$x = 3 \pm 6$$

$$x = -3 \text{ or } x = 9$$

$$\text{Check: } (-3)^2 - 6(-3) = 9 + 18 = 27 \text{ ok}$$

$$9^2 - 6 * 9 = 81 - 54 = 27 \text{ ok}$$

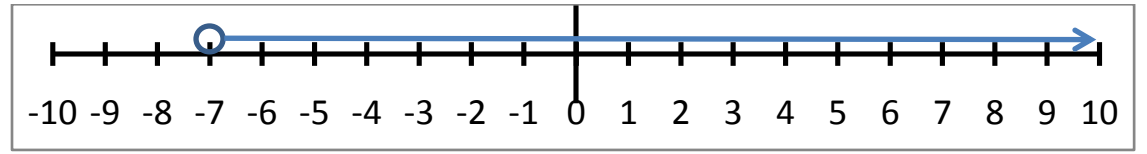
3. Solve for  $x$ , express the result as an interval (round/square brackets) and draw it on a number line. Show your work.

$$-2(x + 3) < 8$$

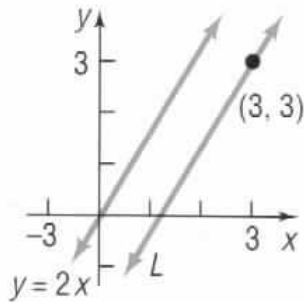
$$x + 3 > \frac{8}{-2} = -4$$

$$x > -7$$

$$(-7, \infty)$$



4. Write an equation of the line  $L$ . Show your work.



$L$  is parallel to  $y = 2x$

The slope of  $y = 2x$  is 2, so the slope of  $L$  is also 2

The equation of  $L$  in slope-intercept form is  $y = 2x + b$

The point  $(3,3)$  is on  $L$  so

$$3 = 2 * 3 + b$$

Solve for  $b$ :

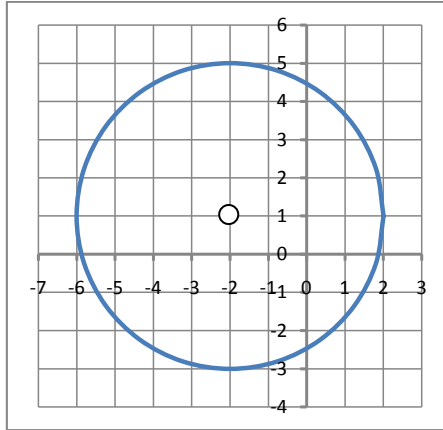
$$3 = 6 + b$$

$$-3 = b$$

The equation of  $L$  is  $y = 2x - 3$

5. Equations of circles:

a. Write an equation of this circle



Center is  $(-2, 1)$ .

Radius is 4

The equation of a circle in standard form is  $(x - h)^2 + (y - k)^2 = r^2$

For this circle,  $(x - (-2))^2 + (y - 1)^2 = 4^2$

That is,  $(x + 2)^2 + (y - 1)^2 = 16$

b. What are the center and radius of a circle with this equation?

$$(x - 3)^2 + y^2 = 5$$

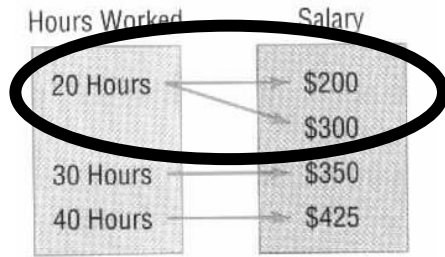
The equation of a circle in standard form is  $(x - h)^2 + (y - k)^2 = r^2$

Center is  $(3, 0)$

Radius is  $\sqrt{5}$

6. Which of the following define  $y$  as a function of  $x$ ? If not, show why not

a.  $x$ : Hours worked,  $y$ : Salary



**NO**

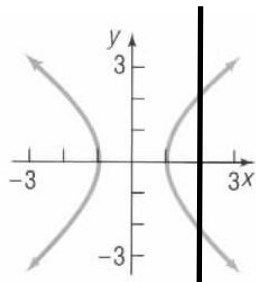
b.  $\{(-2,4), (-1,1), (0,0), (1,1)\}$  **YES**

c.  $y = \frac{1}{x}$  **YES**

d.  $y^2 = 4 - x^2$  **NO**

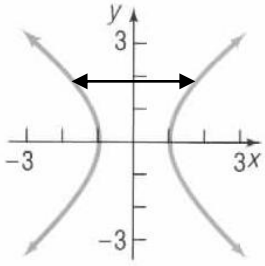
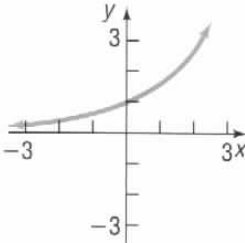
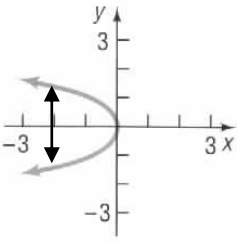
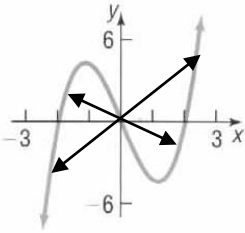
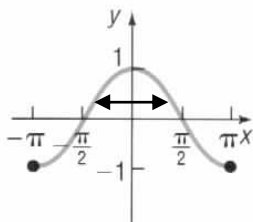
Solve for  $y$ :  $y = \pm\sqrt{4 - x^2}$  which has two values if  $-2 < x < 2$

e.



**NO fails the straight line test**

7. For each graph, which symmetry or symmetries apply? Circle your answer

	<p>X-axis   Y-axis   <b>Origin</b>   None</p>
	<p>X-axis   Y-axis   Origin   <b>None</b></p>
	<p><b>X-axis</b>   Y-axis   Origin   None</p>
	<p>X-axis   Y-axis   <b>Origin</b>   None</p>
	<p>X-axis   <b>Y-axis</b>   Origin   None</p>

8. What is the domain of each of these functions? Show your work.

a.  $g(x) = \frac{x}{x^2-16}$

OK except for denominator = 0

$$x^2 - 16 \neq 0$$

$$(x + 4)(x - 4) \neq 0$$

$$(x + 4) \neq 0 \text{ and } (x - 4) \neq 0$$

$$x \neq 4 \text{ or } -4$$

All  $x$  except 4 and  $-4$

b.  $h(x) = \sqrt{3x - 12}$

OK except  $\sqrt{\phantom{x}} < 0$

$$\text{So } 3x - 12 \geq 0$$

$$3x \geq 12$$

$$x \geq 4$$

$$[4, \infty)$$

9. If  $f(x) = \frac{2x+1}{3x-5}$  then evaluate: (Show your work.)

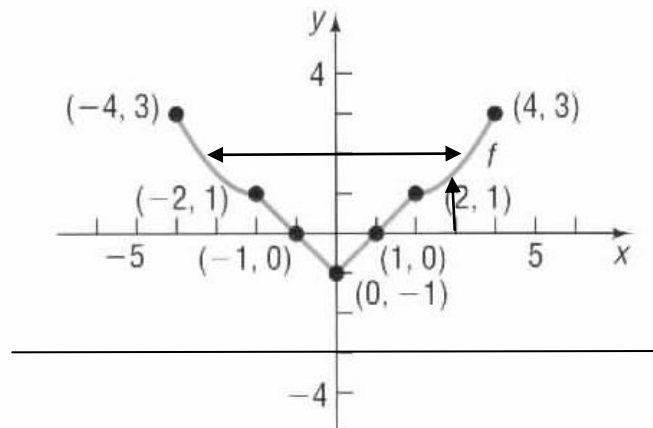
a.  $f(0)$

$$f(0) = \frac{2 * 0 + 1}{3 * 0 - 5} = -\frac{1}{5}$$

b.  $f(-x)$

$$f(-x) = \frac{2*(-x)+1}{3*(-x)-5} = \frac{-2x+1}{-3x-5}$$

10. The graph of  $y = f(x)$  is shown below:



a. Is  $f(x)$  even, odd, or neither?

b. Is  $f(3)$  positive, negative, or zero?

c. How often does the line  $y = -3$  intersect the graph?

11. Calculate:

a.  $(2 - 3i) + (6 + 8i)$

$$(2 - 3i) + (6 + 8i) = (2 + 6) + (-3i + 8i) = 8 + 5i$$

b.  $2i(2 - 3i)$

$$2i(2 - 3i) = 2i * 2 - 2i * 3i = 4i - 6i^2 = 4i - 6(-1) = 6 + 4i$$

12. Does the equation  $x^2 + x + 1 = 0$  have 0, 1, or 2 *real* solutions? Show your work.

$$\text{Discriminant: } D = b^2 - 4ac = 1^2 - 4 * 1 * 1 = -3$$

So there are 0 real solutions



Extra credit - Show your work for all of them.

13. Solve for x:  $\frac{x}{x-2} + 3 = \frac{2}{x-2}$ .

$$\begin{aligned}\frac{x}{x-2} + \frac{3(x-2)}{x-2} &= \frac{2}{x-2} \\ \frac{x}{x-2} + \frac{3(x-2)}{x-2} &= \frac{2}{x-2} \\ \frac{x+3x-6}{x-2} &= \frac{2}{x-2} \\ \frac{4x-6}{x-2} &= \frac{2}{x-2}\end{aligned}$$

Possible solution:

$$\begin{aligned}4x - 6 &= 2 \\ 4x &= 8 \\ x &= 2\end{aligned}$$

Check: if  $x = 2$  then the denominator  $x - 2 = 0$

So there is NO SOLUTION

14. Solve for x.

$$x + \sqrt{x} - 20 = 0$$

Substitute  $u = \sqrt{x}$  - then  $x = u^2$

$$u^2 + u - 20 = 0$$

$$(u + 5)(u - 4) = 0$$

$$u = -5 \text{ or } 4$$

But since  $u = \sqrt{x}$  the solution  $u = -5$  is not possible. This is because  $\sqrt{x}$  is defined to be the **non-negative** square root.

$$\text{If } u = 4 \text{ then } \boxed{x = u^2 = 16}$$

Check:

$$16 + \sqrt{16} - 20 = 16 + 4 - 20 = 0 \text{ ok}$$

15. Calculate  $\frac{6-i}{1+i}$

$$\begin{aligned} &= \frac{6-i}{1+i} * \frac{1-i}{1-i} \\ &= \frac{6-i-6i+i^2}{1+i-i-i^2} \\ &= \frac{6-7i-1}{1-(-1)} = \frac{5-7i}{2} = \frac{5}{2} - \frac{7}{2}i \end{aligned}$$

16. Find all solutions to  $x^2 + x + 1 = 0$ .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4 * 1 * 1}}{2 * 1} \\ &= \frac{-1 \pm \sqrt{-3}}{2} \\ &= \frac{-1 \pm \sqrt{3}\sqrt{-1}}{2} \end{aligned}$$

$$= \frac{-1 \pm \sqrt{3}i}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Solutions are  $x = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$  and  $x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$