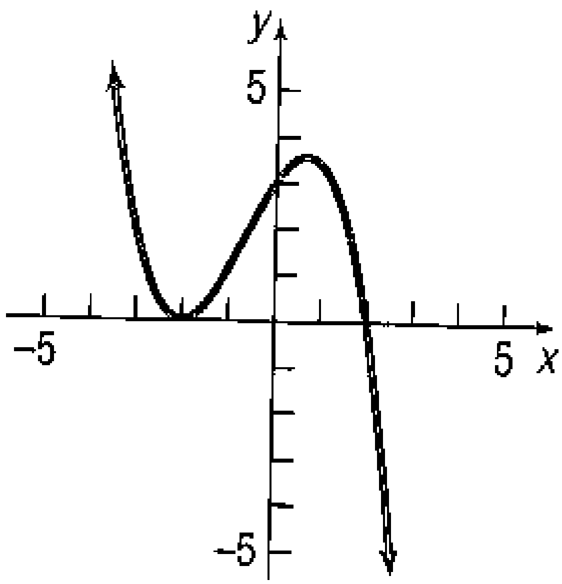
20080128 Practice Test

1. List the intercepts of this equation: (OK just to list)



Solution:

X-intercepts: y=0

(-2,0) and (2,0)

Y-intercepts: x=0

(0,3)

2. Solve for x: $\frac{x}{x^2-9}+\frac{1}{x-3}=0.$
   Common denominator: $x^2-9=(x+3)(x-3)$ which contains $(x-3)$ as a factor.
   $$\frac{x}{x^2-9}+\frac{1}{x-3}\left(\frac{x+3}{x+3}\right)=0$$
   $$\frac{x}{x^2-9}+\frac{x+3}{x^2-9}=0$$
   $$\frac{x+x+3}{x^2-9}=0$$
   $$\frac{2x+3}{x^2-9}=0$$
   $2x+3=0$
   $2x=-3$
   $$x=-\frac{3}{2}$$
   Check: denominator = 0 only if $x=3$ or $x=-3$ so the solution is ok.

3. Solve for x by completing the square
   $x^2+8x-2=0$
   $x^2+8x\quad=2$ The square will be $(x+\frac{8}{2})^2$=$(x+4)^2=x^2+8x+16$
   $x^2+8x+16=2+16$
   $(x+4)^2=18$
   $(x+4)=\pm\sqrt{18}$
   $x=-4\pm\sqrt{18}=-4\pm3\sqrt{2}$ I would accept either one.

4. Solve for x:
   $x^6+3x^3+2=0$
   Target: $u^2+3u+2=0$
   $(x^3)^2+3(x^3)+2=0$

   Substitute $u=x^3$
   Then $x=\sqrt[3]{u}$ and
   $u^2+3u+2=0$

$(u+2)(u+1)=0$
$u=-2\ or\ u=-1$

If $u=-1$
$x=\sqrt[3]{u}=\sqrt[3]{-1}=-1$

If $u=-2$
$x=\sqrt[3]{u}=\sqrt[3]{-2}=\sqrt[3]{-1}\sqrt[3]{2}=-1\sqrt[3]{2}=-\sqrt[3]{2}$

$x=-1\ or\ x=-\sqrt[3]{2}$

Check: if $x=-1$ then $x^6+3x^3+2=(-1)^6+3(-1)^3+2=1+3*(-1)+2=$
$1-3+2=0$ OK
Check: if $x=-\sqrt[3]{2}$ then
$x^6+3x^3+2=\left(-\sqrt[3]{2}\right)^6+3\left(-\sqrt[3]{2}\right)^3+2$
$=(-1)^6*\left(\sqrt[3]{2}\right)^6+3*(-1)^3*\left(\sqrt[3]{2}\right)^3+2$
$=1*2^2+3*(-1)*2+2$
$=4-6+2=0$ OK

5. Solve for x, express the result as an interval, and draw it on a number line.
   $-(2+x)>4$

   Multiply both sides by -1 – and remember to reverse the inequality

   $(2+x)<-4$
   Subtract 2 from both sides

   $x<-6$

   In interval notation

   $(-\infty,-6)$

6. Write an equation of the line parallel to $y = 3x - 1$ and containing the point (2,1).

   Slope of the given line is 3. Slope of parallel line is the same, so $m = 3$.

   One way: use slope intercept form and substitute the known point for x and y

   $y = mx + b$

   $1 = 3 \cdot 2 + b$

   $-5 = b$

   So the equation is $\boxed{y = 3x - 5 \text{ - answer in slope-intercept form}}$

   Another way: use the point-slope form and substitute the known point:

   $$m = \frac{y - y_1}{x - x_1}$$

   $3 = \frac{y-1}{x-2}$ - I would accept this answer since I asked for "an equation"

   Mult by $x - 2$: $3(x - 2) = y - 1$

   Distribute: $3x - 6 = y - 1$

   Add 1: $3x - 5 = y$ or $\boxed{y = 3x - 5}$

7. Equations of circles:
   a. Write an equation of the circle with center at (-1,0) and radius 4.

      $(x - h)^2 + (y - k)^2 = r^2$

      $(x - (-1))^2 + (y - 0)^2 = 4^2$

      $(x + 1)^2 + y^2 = 16$

   b. What are the center and radius of a circle with equation

   $2(x - 3)^2 + 2(y + 1)^2 = 8$?

   Divide by 2 – this is critical!

$$(x-3)^2 + (y+1)^2 = \frac{8}{2} = 4$$
$$h = 3, y = -1, r = \sqrt{4} = 2$$

The center is $(3, -1)$ and the radius is 2

8. Which of the following define y as a function of x? If not, give an example why not
   a. $\{(-2,5), (2,3), (-3,7), (-2,7)\}$ No. -2 is mapped to 2 values

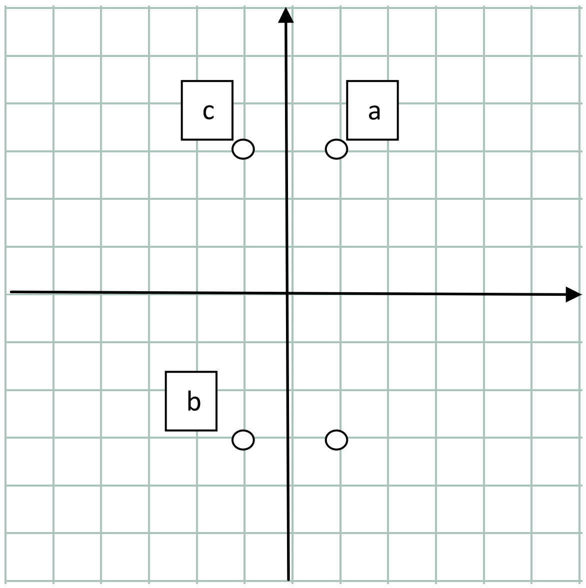
   b. $\{(4,-4), (-3,4), (-1,1), (-6,0)\}$ Yes

   c. $x = y^2$ Solve for y: $y = \pm\sqrt{x}$ so y can have 2 values.

   d. $y = \sqrt{x}$ Yes

   e. $y = x^3 - 13x + 2$ Yes

9. Plot the point (1,-3). Then plot the points that are symmetric to it about
   a. The x-axis (1,-(-3)) = (1,3)
   b. The y-axis (-1,-3)
   c. The origin (-1,-(-3))=(-1,3)



10. What is the domain of $g(x) = \frac{x}{\sqrt{x-2}}$?

    Points NOT in the domain:

    (1) Denominator = 0

    $x - 2 = 0$

    $x = 2$

    (2) Square root of negative number

    $x - 2 < 0$

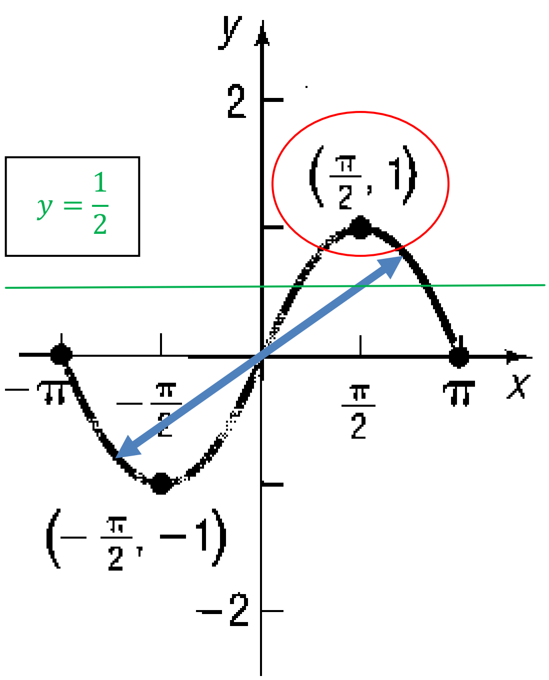
    $x < 2$

    Points that ARE in the domain:

    $x > 2$ or $(2, \infty)$

11. If $f(x) = 2x^2 - 3x$ then evaluate:
    a. $f(-2)$
       $f(-2) = 2(-2)^2 - 3(-2) = 2 \cdot 4 + 6 = 14$
    b. $f(-x)$
       $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$
    c. $f(x-1)$
       $f(x-1) = 2(x-1)^2 - 3(x-1)$
       $= 2x^2 - 4x + 2 - 3x + 3$
       $= 2x^2 - 7x + 5$

12. The graph of $y = f(x)$ is shown below:
    a. Is $f(x)$ even, odd, or neither? ODD – the graph is symmetric through the origin.
    b. What is $f\left(\frac{\pi}{2}\right)$? 1
    c. How often does the line $y = \frac{1}{2}$ intersect the graph? 2



13. Calculate: $\frac{2+3i}{1-i}$

    Key step: multiply numerator and denominator by the *complex conjugate* of the denominator.

    Complex conjugate of $a+bi$ is $a-bi$ so complex conjugate of $1-i$ is $1—i=1+i$.

    Remember that $i^2=-1$

$$\frac{2+3i}{1-i}=\frac{2+3i}{1-i}*\frac{1+i}{1+i}$$
$$=\frac{2+1*3i+2*i+3i^2}{1-i+i-i^2}$$
$$=\frac{2+5i+3*(-1)}{1-(-1)}$$
$$=\frac{-1+5i}{2}$$
$$\boxed{=-\frac{1}{2}+\frac{5}{2}i}$$

14. Find solutions of $x^2-2x+2=0$ (real or otherwise).

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-2)\pm\sqrt{(-2)^2-4*2*1}}{2*1}$$
$$=\frac{2\pm\sqrt{4-8}}{2}$$
$$=\frac{2\pm\sqrt{-4}}{2}$$
$$=\frac{2\pm2\sqrt{-1}}{2}$$
$$=\frac{2\pm2i}{2}$$

$=1\pm i$ Solutions are $x=1+i$ and $x=1-i$

Check:

If $x=1+i$

$$x^2-2x+2=(1+i)^2-2(1+i)+2$$
$$=1+2i+i^2-2-2i+2$$
$$=1+2i-1-2-2i+2$$
$$=(2i-2i)+(1-1-2+2)$$
$$=0+0$$
$$=0$$

If $x = 1 - i$

$$\begin{aligned} x^2 - 2x + 2 &= (1-i)^2 - 2(1-i) + 2 \\ &= 1 - 2i + (-i)^2 - 2 + 2i + 2 \\ &= 1 - 2i - 1 - 2 + 2i + 2 \\ &= (-2i + 2i) + (1 - 1 - 2 + 2) \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

Aside: $(-i)^2 = (-1)^2 * i^2 = 1 * i^2 = 1 * (-1) = -1$