

- Try a substitution to make this a quadratic equation.
Target equation in terms of u is usually patterned after the equation in x
- Substitution to try: focus on the u term in the target equation and look back to the corresponding part of the equation in x . Substitute for u in the target equation and see if you get the original equation (in x)
- Then solve for x in terms of u
- Solve the equation in u .
- Derive solutions for x from the solutions for u .
- Check the solutions

Examples: p. 139

#50: $x^4 - 10x^2 + 25 = 0$

- Target: $u^2 - 10u + 25 = 0$
- Substitution: $u = x^2$. Then $u^2 - 10u + 25 = 0$ is the same as $(x^2)^2 - 10(x^2) + 25 = 0$ or $x^4 - 10x^2 + 25 = 0$. So the substitution works.
- $u = x^2$ - solve for x . Sqrt of both sides: $x = \pm\sqrt{u}$
- Solve $u^2 - 10u + 25 = 0$
 $(u - 5)^2 = 0$
 $u - 5 = \pm\sqrt{0} = 0$
REMEMBER: $\sqrt{0} = 0$ since $0 * 0 = 0$. Also remember: $-0 = 0$.
So $u = 5$ is the single solution in u
- If $u = 5$ then $x = \pm\sqrt{u} = \pm\sqrt{5}$ so the solutions for x are

$x = \sqrt{5}$ and $x = -\sqrt{5}$

- Check: if $x = \sqrt{5}$ then
$$x^4 - 10x^2 + 25 = (\sqrt{5})^4 - 10(\sqrt{5})^2 + 25$$
$$= 5^{(\frac{1}{2}*4)} - 10 * 5^{(\frac{1}{2}*2)} + 25 = 5^2 - 10 * 5^1 + 25 = 25 - 50 + 25 = 0 \text{ OK}$$
And if $x = -\sqrt{5}$ then
$$x^4 - 10x^2 + 25 = (-\sqrt{5})^4 - 10(-\sqrt{5})^2 + 25$$
$$= (-1)^4 * (\sqrt{5})^4 - 10 * (-1)^2 * (\sqrt{5})^2 + 25$$
$$= (\sqrt{5})^4 - 10(\sqrt{5})^2 + 25$$
Just like above so this solution is OK too.

Example:

$$x - \sqrt{x} = 3$$

- a. Target: $u^2 - u = 3$
b. Substitution to try: $u = \sqrt{x}$. Then $u^2 - u = 3$ becomes $(\sqrt{x})^2 - (\sqrt{x}) = 3$ or $x - \sqrt{x} = 3$
c. Then solve for x in terms of u

$$u = \sqrt{x}$$

$$u^2 = (\sqrt{x})^2 = x \text{ so } x = u^2$$

- d. Solve the equation in u .

$$u^2 - u = 3$$

$$u^2 - u - 3 = 0$$

$$\text{Quadratic formula: } u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{1 \pm \sqrt{13}}{2}$$

$$\text{So solutions in } u \text{ are } u = \frac{1 - \sqrt{13}}{2} \approx -1.30 \text{ and } u = \frac{1 + \sqrt{13}}{2} \approx 2.30$$

- e. Derive solutions for x from the solutions for u .

$$\text{If } u = \frac{1 - \sqrt{13}}{2} \text{ then } x = u^2 = \left(\frac{1 - \sqrt{13}}{2}\right)^2 \approx 1.69$$

$$\text{If } u = \frac{1 + \sqrt{13}}{2} \text{ then } x = u^2 = \left(\frac{1 + \sqrt{13}}{2}\right)^2 \approx 5.30$$

$$\text{Check the solutions: } x - \sqrt{x} = 3$$

$$x = 1.69 \text{ then } 1.69 - \sqrt{1.69} \approx 0.39 \neq 3 \text{ so this solution does NOT check.}$$

$$x = 5.30 \text{ then } 5.30 - \sqrt{5.30} \approx 2.998 \approx 3 \text{ so this solution looks good}$$

Explanation:

Why was the first solution NOT good? Go back to the solutions for u :

$$u = \frac{1 - \sqrt{13}}{2} \approx -1.30 \text{ and } u = \frac{1 + \sqrt{13}}{2} \approx 2.30$$

Remember $u = \sqrt{x}$ which is *defined to be non-negative*. So the first solution is not possible. You might remember to look for such things, but you might not, so go ahead and check the solutions in x in the original equation, like I did.