

APPLICATIONS

20. Banking Wendy, a loan officer at a bank, has \$1,000,000 to lend and is required to obtain an average return of 18% per year. If she can lend at the rate of 19% or at the rate of 16%, how much can she lend at the 16% rate and still meet her requirement?

Quantities

Total amount loaned	A	\$	1,000,000
Amount loaned at 19%	A_{19}	\$	
Amount loaned at 16%	A_{16}	\$	Solution needed – this is what you are being asked
Required total amount earned	M	\$	
Amount earned at 19%	M_{19}	\$	
Amount earned at 16%	M_{16}	\$	
Overall rate of return	r		18%

What you know to start

$$A = A_{19} + A_{16}$$

$$A = 1,000,000$$

$$M = M_{19} + M_{16}$$

$$M_{19} = 0.19A_{19}$$

$$M_{16} = 0.16A_{16}$$

$$r = 0.18$$

$$r = \frac{M}{A}$$

Steps to solution: goal is to get an equation for A_{16}

$$r = \frac{M}{A} \text{ means } M = rA = 0.18 * 1,000,000 = 180,000$$

$$\text{Then } M = M_{19} + M_{16} \text{ means } 180,000 = M_{19} + M_{16} = 0.19A_{19} + 0.16A_{16}$$

$$A = A_{19} + A_{16} \text{ means } A_{19} = A - A_{16} = 1,000,000 - A_{16} \text{ so}$$

$$180,000 = (1,000,000 - A_{16}) * 0.19 + A_{16} * 0.16$$

Solve this equation for A_{16}

$$180,000 = 190,000 - 0.19A_{16} + 0.16A_{16} = 190,000 - 0.03A_{16}$$

$$180,000 - 190,000 = -0.03A_{16}$$

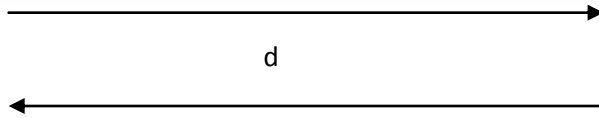
$$-10,000 = -0.03A_{16}$$

$$A_{16} = -\frac{10,000}{-0.03} = 333.333 \text{ She can lend } \$333,333 \text{ at } 16\%$$

22. Business: Blending Coffee A coffee manufacturer wants to market a new blend of coffee that sells for \$3.90 per pound by mixing two coffees that sell for \$2.75 and \$5 per pound, respectively. What amounts of each coffee should be blended to obtain the desired mixture?

[Hint: Assume that the total weight of the desired blend is 100 pounds.]

26. Physics: Uniform Motion A motorboat heads upstream on a river that has a current of 3 miles per hour. The trip upstream takes 5 hours, and the return trip takes 2.5 hours. What is the speed of the motorboat? (Assume that the motorboat maintains a constant speed relative to the water.)



Quantities

d	Distance (one direction)	miles	
v_u	Velocity upstream	Miles/hour	
v_d	Velocity downstream	Miles/hour	
v_b	Velocity of the boat in still water	Miles/hour	
t_u	Time of upstream trip	Hours	5
t_d	Time of downstream trip	Hours	2.5

What you know:

$$\text{distance} = \text{velocity} * \text{time}$$

$$d = v_u * t_u = v_d * t_d$$

The current adds or subtracts to the boat's velocity as the boat goes upstream or downstream.

$$v_u = v_b - 3 \text{ and } v_d = v_b + 3$$

Substitute these and the known trip times into $v_u * t_u = v_d * t_d$

$$(v_b - 3) * 5 = (v_b + 3) * 2.5$$

Solve for v_b - start by distributing

$$5v_b - 15 = 2.5v_b + 7.5$$

$$5v_b - 2.5v_b = 7.5 + 15$$

$$2.5v_b = 22.5$$

$$v_b = \frac{22.5}{2.5} = 9$$

The boat's speed is 9 miles per hour

30. Working Together on a Job Patrice, by himself, can paint four rooms in 10 hours. If he hires April to help, they can do the same job together in 6 hours. If he lets April work alone, how long will it take her to paint four rooms?


Approach:

Key starting point: work in terms of rate of work = $\frac{\text{rooms painted}}{\text{time worked}}$ with units "rooms painted per hour". Then you can define a rate of work for Patrice, another for April, and when they work together their rate of work is the SUM of the 2 rates.

INTERVAL NOTATION and SOLVING INEQUALITIES

Interval	Inequality	Graph
The open interval (a, b)	$a < x < b$	
The closed interval $[a, b]$	$a \leq x \leq b$	
The half-open interval $[a, b)$	$a \leq x < b$	
The half-open interval $(a, b]$	$a < x \leq b$	
The interval $[a, \infty)$	$x \geq a$	
The interval (a, ∞)	$x > a$	
The interval $(-\infty, a]$	$x \leq a$	
The interval $(-\infty, a)$	$x < a$	
The interval $(-\infty, \infty)$	All real numbers	

INTERVAL	INEQUALITY	GRAPH on NUMBER LINE
$(-1,3)$	$-1 < x < 3$	
$(0,4]$	$0 < x \leq 4$	
$(-2, \infty)$	$-2 < x < \infty$	<p>or</p>
$(-\infty, 4]$	$x \leq 4$	

INTERVAL	INEQUALITY	GRAPH on NUMBER LINE
$(-\infty, -1)$	$x < -1$	
$[1, 4)$	$1 \leq x < 4$	
$[2, 10)$	$10 > x \geq 2$ Rewrite as $2 \leq x < 10$	

SOLVING INEQUALITIES BY ALGEBRA

ACTION	EFFECT Starting from $A < B$ (similar for $A \leq B$)
Add or subtract a finite number or expression to both sides	Same direction $A + 10 < B + 10$
Simplify both sides	Same direction
Multiply (or divide) both sides by a <i>positive</i> finite number	Same direction $\frac{1}{2}A < \frac{1}{2}B$
Swap sides	Reverse direction $B > A$
Multiply both sides by a <i>negative</i> finite number or expression	Reverse direction $-A > -B$
Invert both sides (if neither is zero)	Reverse direction $\frac{1}{A} > \frac{1}{B}$
Add or subtract ∞ to both sides	Not defined!
Multiply both sides by $0, \infty, \text{ or } -\infty$	Not defined!
Invert both sides (if at least one is zero)	Not defined!

Examples of *linear inequalities*

Find the solution set for $x - 6 < 1$. Express the result in all 3 forms (interval, inequality, and graph on number line).

Add 6	$x < 1 + 6$
	$x < 7$
Interval form:	$(-\infty, 7)$

Solve $2 - 3x \leq 5$.

Add $3x$	$2 \leq 5 + 3x$
Subtract 5	$-3 \leq 3x$
Divide 3	$-1 \leq x$
Swap sides	$x \geq -1$
Interval form	$[-1, \infty)$

Solve $4 \leq 2x + 2 \leq 10$

Subtract 2	$2 \leq 2x \leq 10$
Divide 2	$1 \leq x \leq 5$
Interval form	$[1,5]$

Solve $3x + 4 > \frac{1}{3}(x - 2)$

Solve $|x| < 3$

Algebra – remember $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

When $x \geq 0$: $x < 3$ so $0 \leq x < 3$

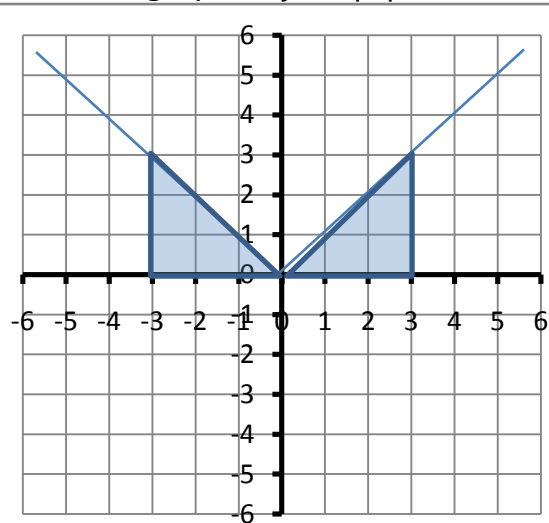
When $x < 0$: $-x < 3$

Multiply by -1 remember this reverses the inequality: $x > -3$ so $-3 < x < 0$

Put the two pieces together:

$-3 < x < 3$ or $(-3, 3)$

Sketch the graph of $y = |x|$



Solve $|x + 4| < 2$

Either: $0 \leq x + 4 < 2$ or $0 < -(x + 4) < 2$

When $0 \leq x + 4 < 2$

$-4 \leq x < -2$

When $0 < -(x + 4) < 2$

$0 < -x - 4 < 2$

$4 < -x < 6$

$-4 > x > -6$

$-6 < x < -4$

So the solution is the union of

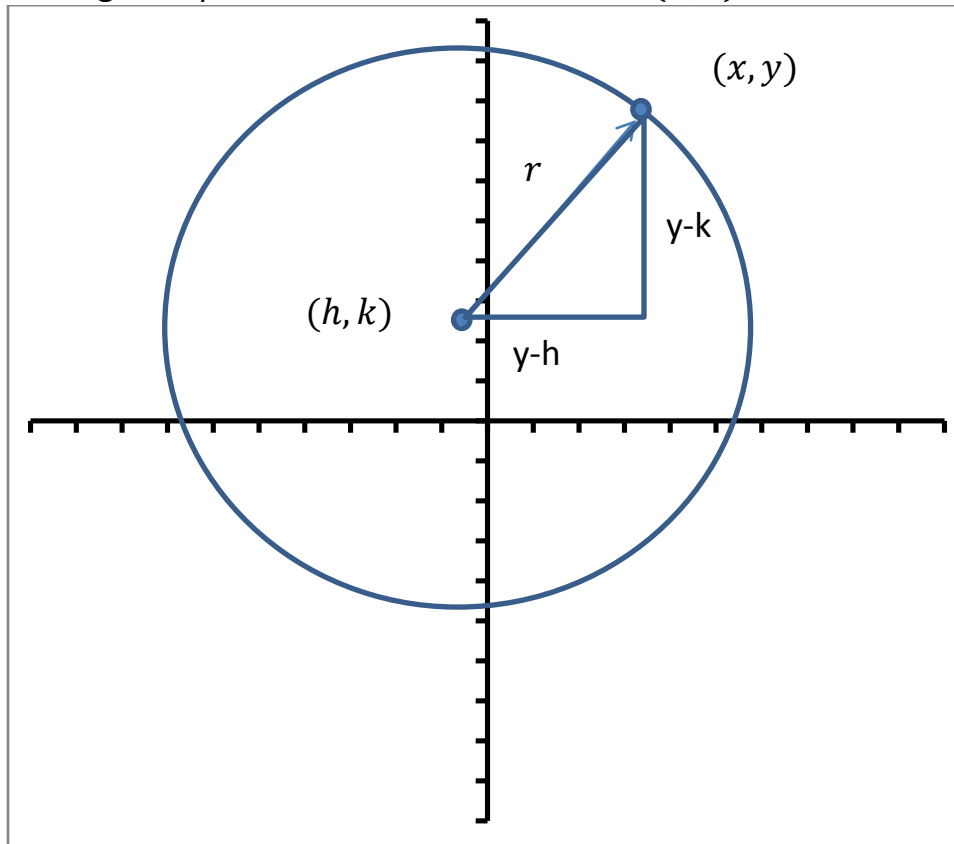
$-4 \leq x < -2$ and $-6 < x < -4$

Which is $-6 < x < -2$ or $(-6, -2)$

Solve $x^2 \geq 16$

CIRCLES

Finding an equation of a circle with center (h, k) and radius r



What does r mean? Distance from center to any point on the circle

Can you calculate it in terms of (h, k) and (x, y) ?

What famous ancient person comes to mind? Pythagoras...

$$r^2 = (x - h)^2 + (y - k)^2$$

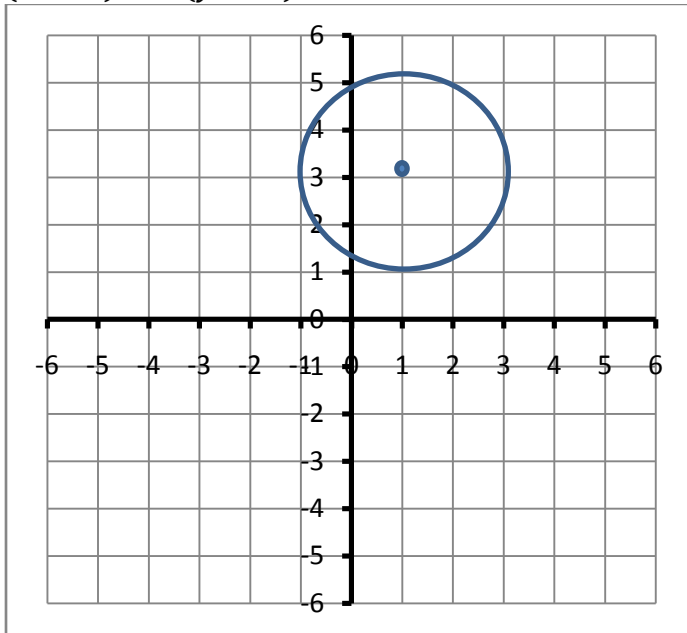
Standard form of the equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

What is the equation of this circle? Center is (1,3) and radius is 2, so

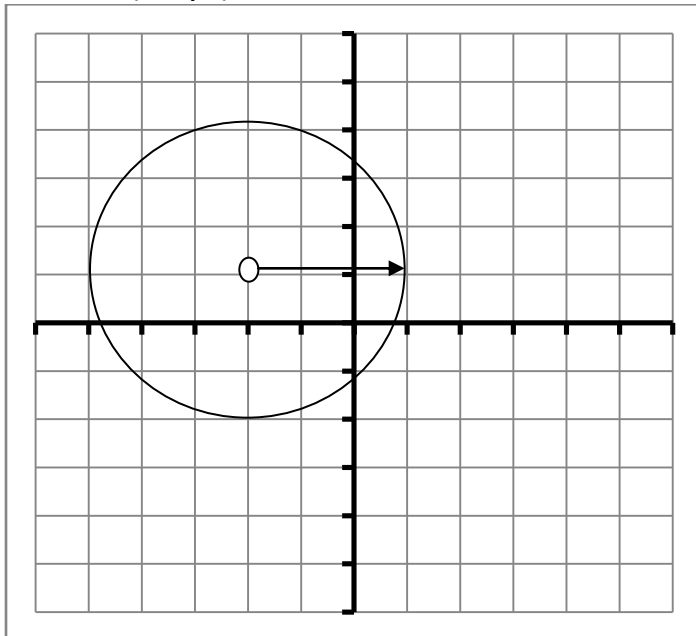
$h = 1, y = 3, r = 2$ and

$$(x - 1)^2 + (y - 3)^2 = 2^2 = 4$$



What circle has equation $(x + 2)^2 + (y - 1)^2 = 9$? Draw and label.

$h = -2$ (why?) and $k = 1. r^2 = 9$ so $r = \sqrt{9} = 3$



What circle has equation $x^2 + (y - 3)^2 = 100$?

$h = 0$ (why?) and $k = 3$; $r^2 = 100$ so $r = 10$

The circle has center $(0,3)$ and radius 10.

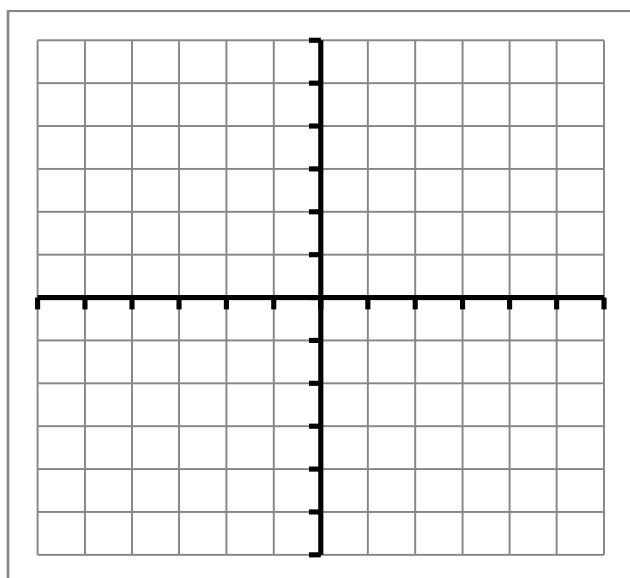
PUTTING EQUATIONS OF CIRCLES INTO STANDARD FORM

Start with this equation: $(x + 2)^2 + (y - 1)^2 = 9$, multiply out, and arrange so that the right side is $= 0$

Put the following equation in standard form and find the center and radius

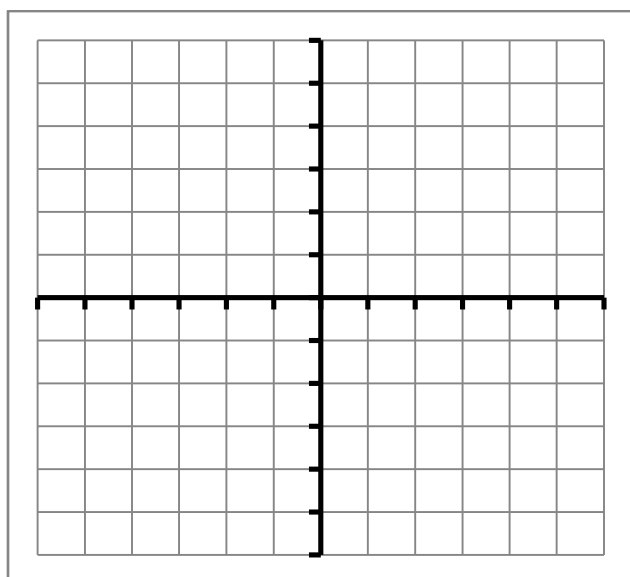
$$x^2 - 6x + 9 + y^2 + 2y + 1 = 25$$

Put this equation into standard form and find the center and radius
 $x^2 + y^2 - 4x - 32 = 0$



Put this equation into standard form and find the center and radius

$$x^2 + y^2 - 6x + 2y + 9 = 0$$



FINDING INTERCEPTS OF CIRCLES

Find all the intercepts of $x^2 + y^2 - 6x + 2y + 6 = 0$ and label them on the plot (previous page)

X-intercepts: set $y = 0$ and solve for:	Y-intercepts:
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