APPLICATIONS

(P. 122, #82): A rectangular window has area=306 sq. cm. and its length is 1 cm greater than its width. What are the dimensions?

- What quantities are involved? Define variables and units. Draw a diagram if that's useful.
- What are you being asked?
- What information are you given about the quantities? Write that information in terms of the variables (write equations)
- Solve the equation(s)
- Write the answer in words, and remember to include the units.

P. 123, #94:

94. Dimensions of a Patio A contractor orders 8 cubic yards of premixed cement, all of which is to be used to pour a patio that will be 4 inches thick. If the length of the patio is specified to be twice the width, what will be the patio dimensions? (1 cubic yard = 27 cubic feet)

COMPLEX NUMBERS

$x^2 = -25$ does not have a solution using real numbers.

Define: $\boldsymbol{i = \sqrt{-1}}$
("i" was used because it is an ***imaginary number***)

Then $i^2 =$
$(2i)^2 =$
and $(-2i)^2 =$

Solving quadratic problems using i:
$x^2 = -25$
$x = \pm\sqrt{-25} = \pm\sqrt{25 \cdot -1} = \pm\sqrt{25}\sqrt{-1} = \pm 5i$

Exercise: solve $x^2 + 9 = 0$

Exercise: solve $x^2 + 2x + 2 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 1} =$$

Numbers like $1 + i$, $3 - 2i$, $a + bi$ are called ***complex numbers.***
If $z = a + bi$ then
a is the ***real part*** of z and b is the ***imaginary part*** of z

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

Digression: "if and only if" is the same as:
"is equivalent to", "iff", "↔"

If the discriminant $b^2 - 4ac < 0$ then the quadratic equation has no *real* solutions but it will have 2 *complex* solutions.

If you are asked to find *real* solutions, and there aren't any, then you can say "there are none."
If you are asked to solve "in the complex number system" or "over the complex numbers", or you are not told specifically to find real solutions, then provide the solutions – whether they are real numbers or not

A real number IS a complex number, whose imaginary part = ??

Exercise: solve $x^2 - 2x + 5 = 0$

Complex arithmetic

Adding (or subtracting) complex numbers: add (or subtract) their real and imaginary parts separately

$(2+3i)+(4-i)=(\qquad + \qquad i)$

Exercises:

$(4+5i)+(-8+2i)=$

$(3-4i)-(-3-4i)=$

Multiplication: distribute (FOIL)

$(1+2i)(2-3i)= \qquad + \qquad i\ + \qquad i \qquad\qquad (i^2)$

$=$

Exercise:
$(5+3i)(2-i)$

Complex conjugate:
If $z = a + bi$ then its conjugate is $\bar{z} = \overline{a+bi} = a - bi$

Examples:

z	$\bar{z}$
$2+i$	
$3-2i$	
i	
$-4+3i$	

Theorem: if $z = a + bi$ then $z\bar{z} = a^2 + b^2$ – which is a *real* number
(see p. 127 of the book for a proof)

Example: if $z = 3 + 4i$ then

$z\bar{z} =$

Dividing complex numbers:

- Express as a fraction (if not already)
- Multiply numerator and denominator by the complex conjugate of the *denominator* (this amounts to multiplying the fraction by 1)

Example:

$$(3+2i) \div (2-i) = \frac{3+2i}{2-i} = \frac{3+2i}{2-i} \cdot \frac{\mathbf{2}+\boldsymbol{i}}{\mathbf{2}+\boldsymbol{i}}$$

$$= \frac{(3+2i)(2+i)}{2^2+1^2} =$$

Exercise:

Powers of i

i^1		i
i^2		-1
i^3	$i \cdot i^2 =$	
i^4	$(i^2)(i^2) =$	
i^5	$i^4 i^1 =$	
i^6	$i^4 i^2 =$	

Exercise: calculate i^{13}

~~Exercise: find all solutions to $x^4 + 1 = 0$~~ **I GOOFED**

Exercise: find all solutions to $x^4 - 1 = 0$

$$x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1)$$

Either $x = 1, x = -1$, or $x^2 + 1 = 0$

Solving $x^2 + 1 = 0$

$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

So solutions are $x = 1, x = -1, x = i, x = -i$

RADICAL EQUATIONS

A radical equation includes roots or fractional exponents.
Strategy: rearrange conveniently and remove the radical. And check all solutions!

Example: $\sqrt[3]{1-2x}-1=0$

Add 1	$\sqrt[3]{1-2x}=1$
Cube	$1-2x=1^3=1$
Subtract 1	$-2x=1-1=0$
Divide -2	$x=0$ (solution)
Check:	$\sqrt[3]{1-2*0}-1=\sqrt[3]{1}-1=1-1=0$ Solution is ok.

Example: $\sqrt{12-x}-x=0$

Add x	$\sqrt{12-x}=x$
Square	$12-x=x^2$
Subtract $(12-x)$ and swap	$x^2+x-12=0$
Factor	$(x+4)(x-3)=0$
Solutions	$x=-4\ and\ x=3$
Check $x=-4$	$\sqrt{12-(-4)}-(-4)=\sqrt{16}+4=8\neq 0$ This is not a correct solution – it's called *extraneous*
Check $x=3$	$\sqrt{12-3}-3=\sqrt{9}-3=0$ Solution is ok

What happened? Go back to
$\sqrt{12-x}=x$
This implies that $x\geq 0$ since this $\sqrt{\ }$ is defined to be the *positive* square root. So $x=-4$ could not be a solution.
Just for fun - suppose the original equation were $-\sqrt{12-x}-x=0$.

Example: $x^{\frac{1}{4}} = -1$

Raise to 4th power	$x = (-1)^4 = 1$

Substitution (to create a quadratic equation)

Example: $2x^4 - 5x^2 - 12 = 0$

Target: $2u^2 - 5u - 12 = 0$

Substitution: $u = x^2$ and then $x = \pm\sqrt{u}$. **It's important to remember that x can be either** $\sqrt{u}$ or $-\sqrt{u}$.

$2x^4 - 5x^2 - 12 = 2(x^2)^2 - 5(x^2) - 12 = 2u^2 - 5u - 12 = 0$

Solve quadratic equation (in u):

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 * 2 * (-12)}}{2 * 2}$$
$$= \frac{5 \pm \sqrt{25 - (-96)}}{4} = \frac{5 \pm \sqrt{121}}{4} = \frac{5 \pm 11}{4}$$

Solutions: $u = \frac{16}{4} = 4 \; or \; u = -\frac{6}{4} = -\frac{3}{2}$

Substitute back - to get the solution(s) in x.

$u = 4$ $x = \pm\sqrt{u} = \pm\sqrt{4} = \pm 2$	$u = -\dfrac{3}{2}$ $x = \pm\sqrt{u} = \pm\sqrt{-\frac{3}{2}}$ which is not defined

Solutions: $x = 2 \; or \; x = -2$

Check all the solutions!

$x = 2$: $2 * 2^4 - 5 * 2^2 - 12 = 2 * 16 - 5 * 4 - 12 = 32 - 20 - 12 = 0$ OK

$x = -2$: $2 * (-2)^4 - 5 * (-2)^2 - 12 = 2 * 16 - 5 * 4 - 12 = 32 - 20 - 12 = 0$ OK

Example: $x + \sqrt{x} = 6$
First step: $x + \sqrt{x} - 6 = 0$
Target: $u^2 + u - 6 = 0$
Substitution: $u = \sqrt{x}$ so then $x = u^2$ and $x + \sqrt{x} - 6 = u^2 + u - 6 = 0$
Solve quadratic equation (in u)
Factor: $(u + 3)(u - 2) = 0$
$u = -3 \; or \; u = 2$

Substitute back - to get the solution(s) in x.

$u = -3$ $x = u^2 = (-3)^2 = 9$	$u = 2$ $x = u^2 = 2^2 = 4$

Check all the solutions

$x = 9$ $9 + \sqrt{9} = 9 + 3 = 12 \neq 6$ Not a solution. This was based on $u = -3$ and remember $u = \sqrt{x}$ was defined to be the *positive* square root	$x = 4$ $4 + \sqrt{4} = 4 + 2 = 6$ OK

The only solution is $x = 4$.

ABSOLUTE VALUE

$|-2| = 2$ $|3| = 3$ $|0| = 0$

Meaning: $|x|$ is the distance from x to 0 on the number line.

Formula: $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Solving an equation involving absolute values
Example:
$|x| = 2$

Either $x = 2$	or $-x = 2 \rightarrow x = -2$

Solutions: $x = 2$, $x = -2$

Exercise: solve
$|3x - 1| = 2$

Either $3x - 1 = 2$ $3x = 2 + 1 = 3$ $x = 1$	or $3x - 1 = -2$ $3x = -2 + 1 = -1$ $x = -\frac{1}{3}$

Solutions: $x = 1$, $x = -\frac{1}{3}$

Exercise: solve
$|x^{27} + x^{19} - 341| = -2$

$|anything| \geq 0$ so there is no solution.