

ALGEBRA PRACTICE

Algebraic tools for solving equations

- Swap sides
- Eliminate parentheses
- Collect like terms
- Add or subtract the same thing from both sides
- Multiply or divide both sides by the same (non-zero) thing
- Add 0 to something
- Multiply something by 1 → Multiply numerator and denominator of a fraction by the same (non-zero) thing
- Find and use a common denominator

Solve each of these equations:

$$2x + 9 = 5x$$

$$1 - \frac{1}{2}x = 6$$

$$5y + 6 = -18 - y$$

$$3(2 - x) = 2x - 1$$

$$\frac{4}{y} - 5 = \frac{5}{2y}$$

$$(x + 2)(x - 3) = (x + 3)^2$$

$$\frac{2x}{x + 3} = -\frac{6}{x + 3} - 2$$

$$-\frac{4}{x + 4} = -\frac{3}{x + 6}$$

$$A = P(1 + rt) - \text{solve for } r$$

General approach to word problems

- What quantities are involved? Define variables and units.
- What are you being asked?
- What information are you given about the quantities? Write that information in terms of the variables (write equations)
- Solve the equation(s)
- Write the answer in words, and remember to include the units.

Finance A total of \$10,000 is to be divided between Sean and George, with George to receive \$3000 less than Sean. How much will each receive?

QUANTITIES

UNITS

SYMBOL

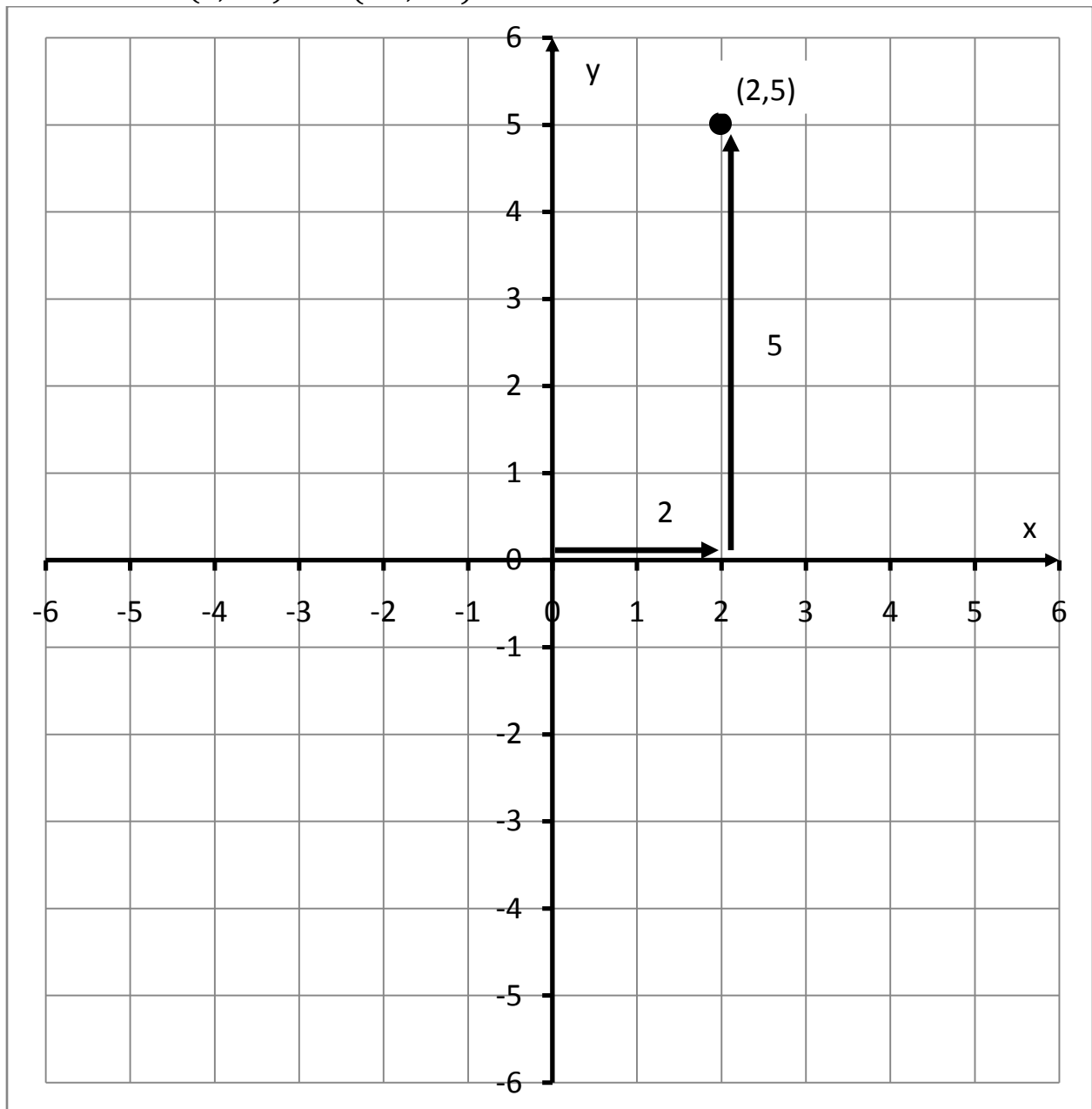
Personal Finance: Cost of a Car The suggested list price of a new car is \$24,000. The dealer's cost is 85% of list. How much will you pay if the dealer is willing to accept \$100 over cost for the car?

Rectangular plane coordinates

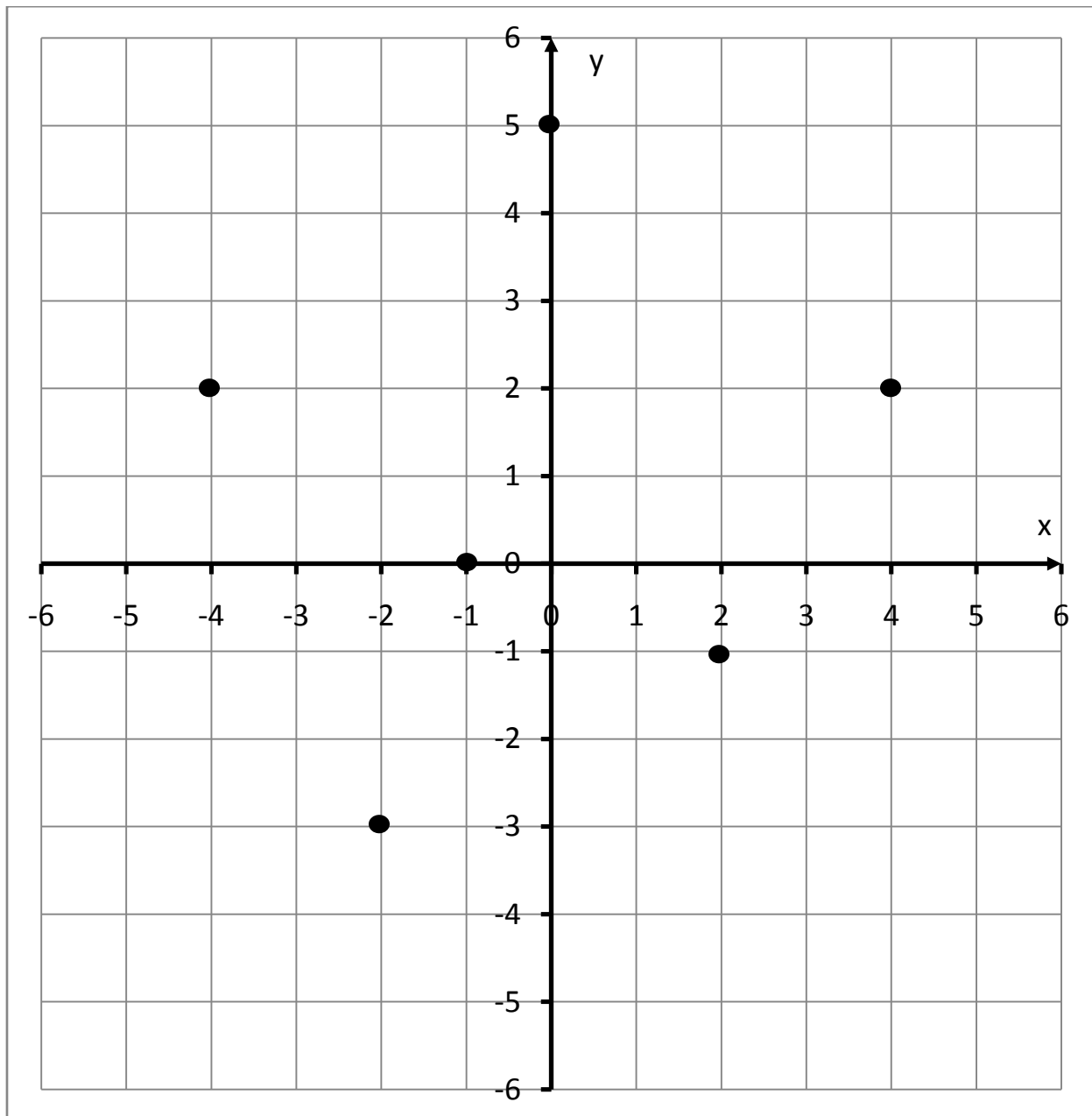
Rectangular coordinates (also called *Cartesian*)

- Label each point with coordinates (x, y)
- $(0,0)$ is the *origin*.
- x is distance from the origin along the horizontal axis (positive to the right, negative to the left)
- y is distance along the vertical axis (positive upward, negative downward)
- Horizontal axis is the x axis, vertical axis is the y axis

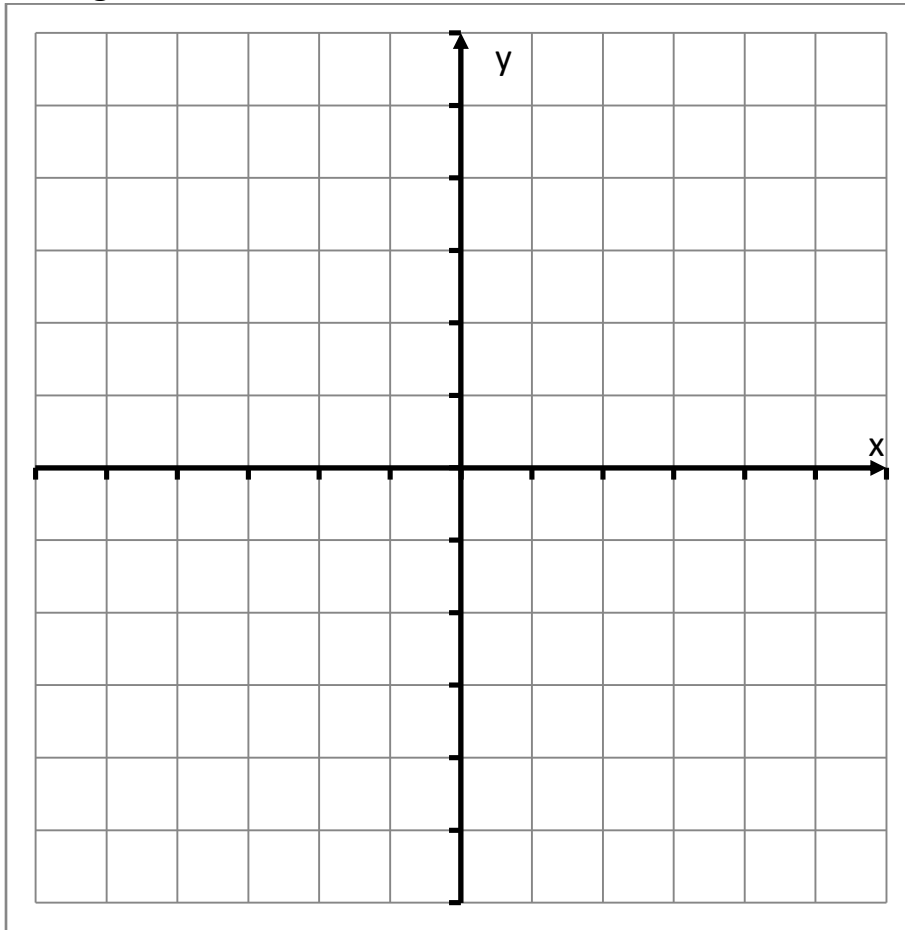
Exercise: Plot $(3, -2)$ and $(-5, -4)$



Exercise: Label these points with their rectangular coordinates



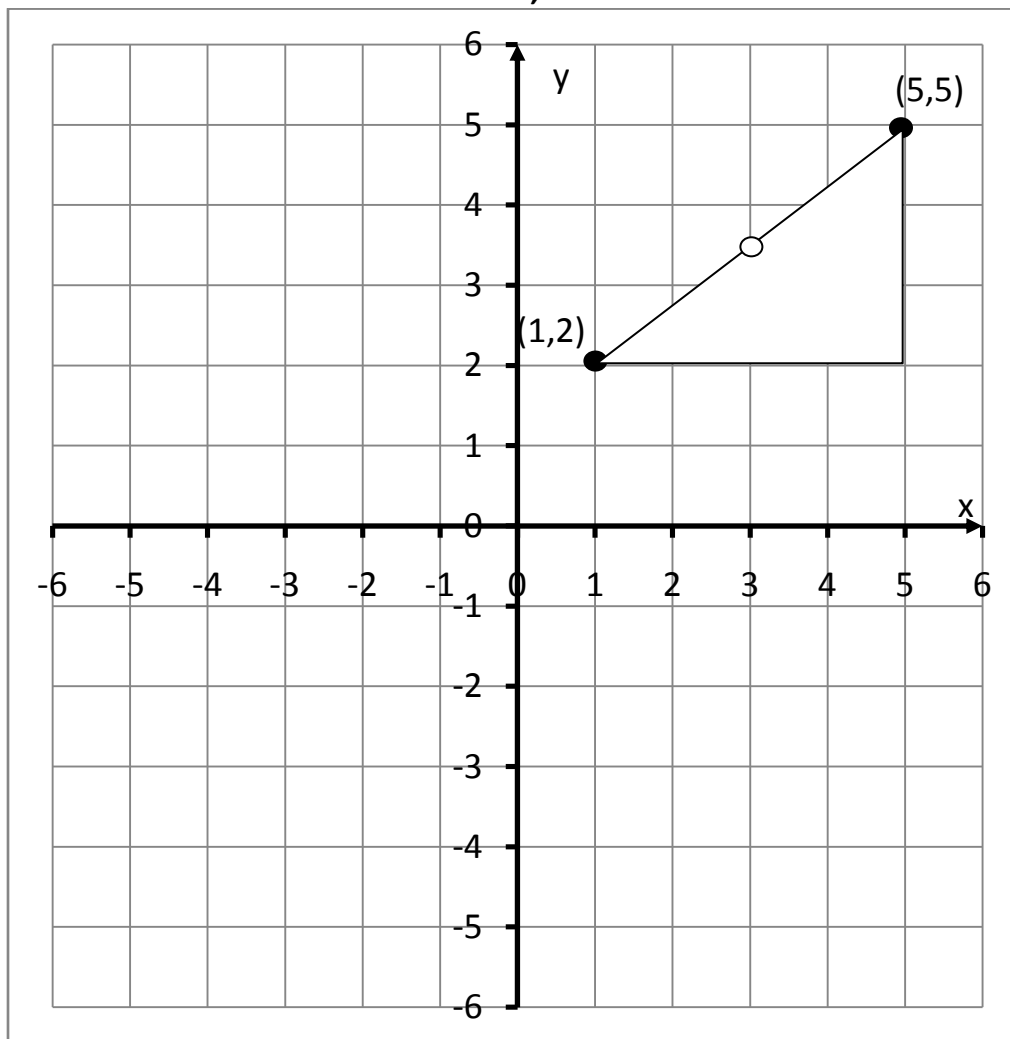
Scaling the axes



Exercise: Plot the point $(52, -18)$

Start: How many units should each square represent? Label the tick marks

DISTANCE BETWEEN TWO POINTS; MIDPOINT BETWEEN TWO POINTS



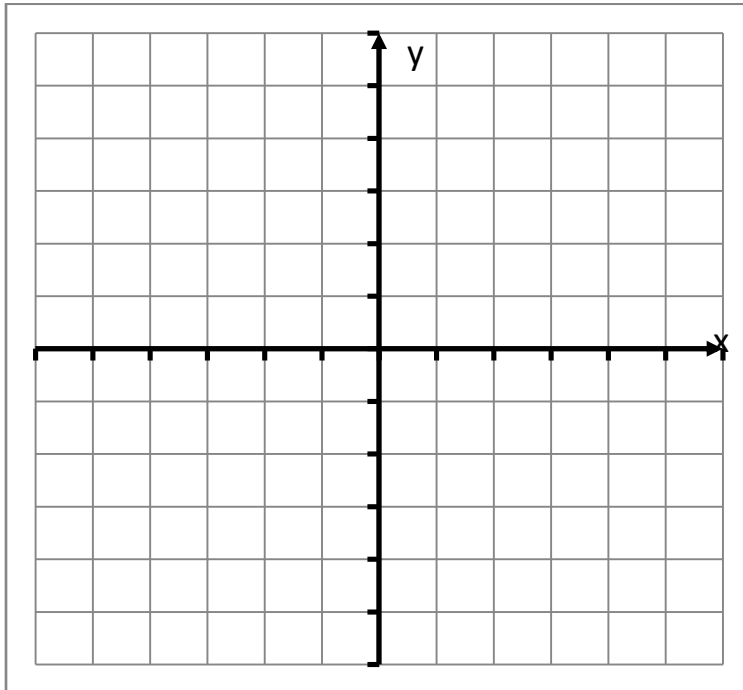
Find the distance between (1,2) and (5,5)

- Distance in horizontal direction:
- Distance in vertical direction:
- Length of diagonal (hypotenuse):

Distance between (x_1, y_1) and (x_2, y_2) : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example problem:



Exercise: Plot the points $(-2,1)$ and $(4,-5)$ and find

- The distance between them
- The midpoint between them

GRAPH OF AN EQUATION

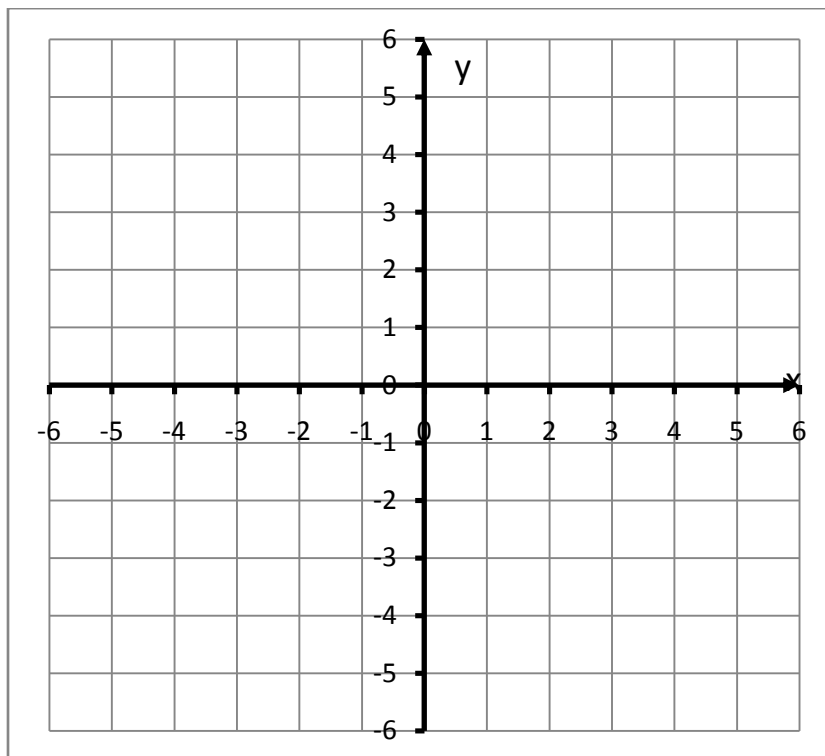
Example: $y = 2x - 1$

$x =$	$y =$
-1	
0	
1	
2	

These are some points that *satisfy* the equation $y = 2x - 1$.

Does the point (3,4) satisfy the equation $y = 2x - 1$?

Test by substituting 3 for x and 4 for y . Is the result a true statement?



The graph of an equation (for example $y = 2x - 1$) is *all* the points that satisfy the equation.

Exercise: if $y = x^2$

1. Which of the following points satisfy the equation?

(4,2)

(2,4)

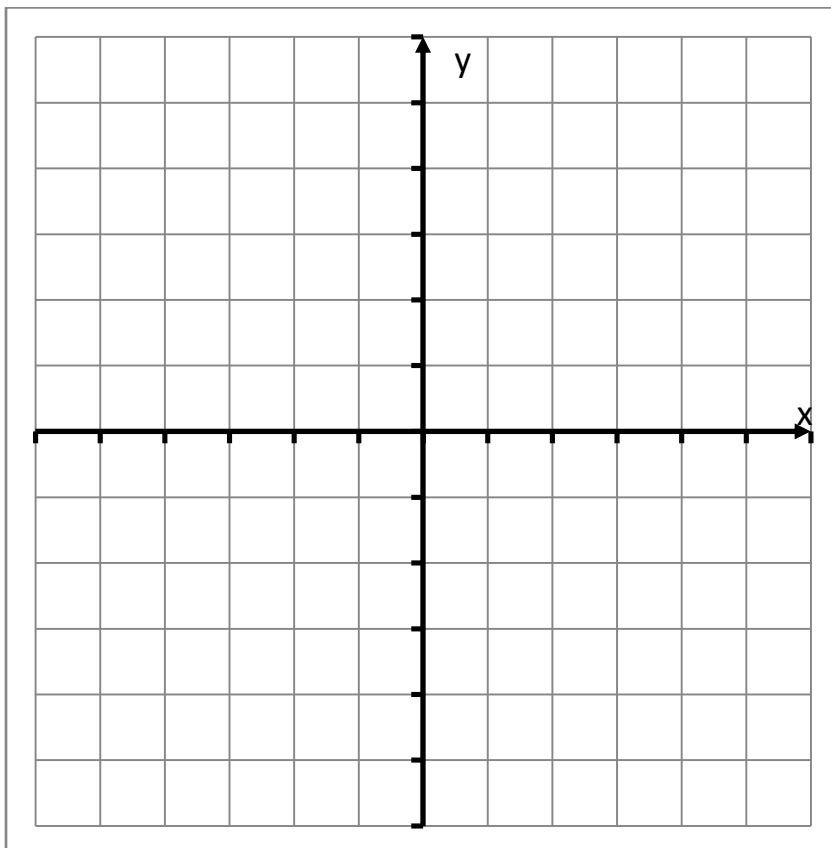
(3,6)

(-5,-25)

(-5,25)

2. Plot at least 5 points that satisfy the equation, and sketch the graph of the equation

x	y



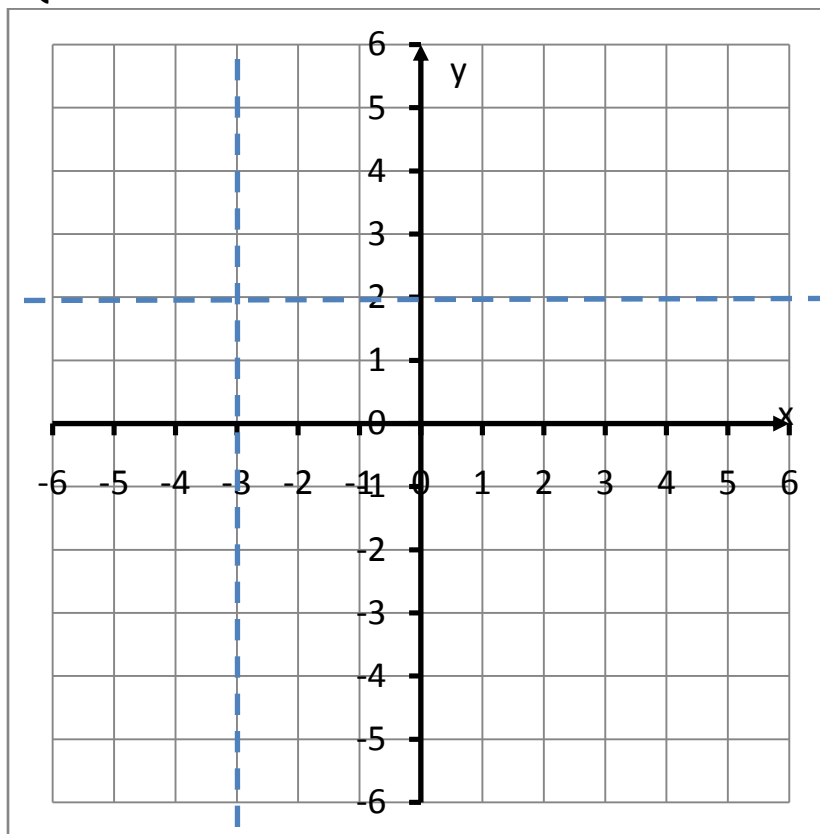
3. Use a graphical calculator to plot the graph of $y = x^2$. Include values of x from -20 to 20.

Steps:

- Enter the equation
- Set the window
 - Xmin, Xmax
 - Ymin, Ymax

- Graph
- Examine the table of values

EQUATIONS OF HORIZONTAL AND VERTICAL LINES

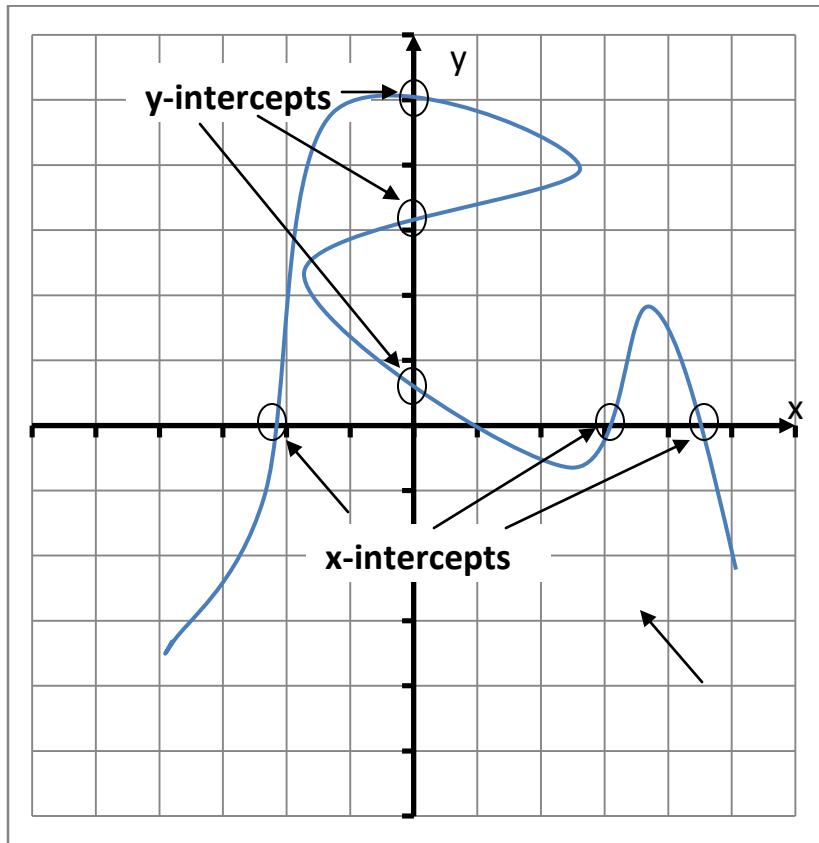


The 2 dashed lines are graphs of (very simple) equations. What are they?

- Horizontal line:
- Vertical line:
-

Exercise: Draw and label the graphs of $x = 4$, $y = -3$, and $x = 0$ on the graph paper above

INTERCEPTS



	Definition in terms of the graph	How to find from the equation	Coordinates:
x-intercepts	Points where the graph intersects the x-axis	Set $y = 0$ and solve for x	$(x\text{-intercept}, 0)$
y-intercepts	Points where the graph intersects the y-axis	Set $x = 0$ and solve for y	$(0, y\text{-intercept})$

Exercise: find the intercepts of $y = 2x - 1$

X-intercepts:

Y-intercepts:

Exercise: Find the intercepts of $x^2 - y - 1 = 0$

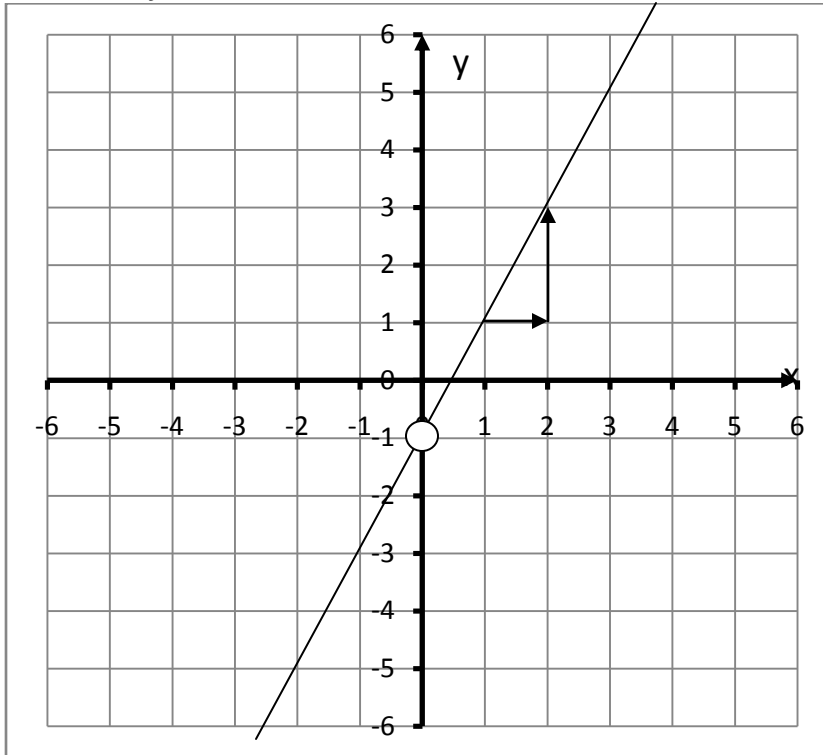
X-intercepts:

Y-intercepts:

LINEAR EQUATIONS

A linear equation in x and y is one whose graph is a straight line.

Example: $y = 2x - 1$



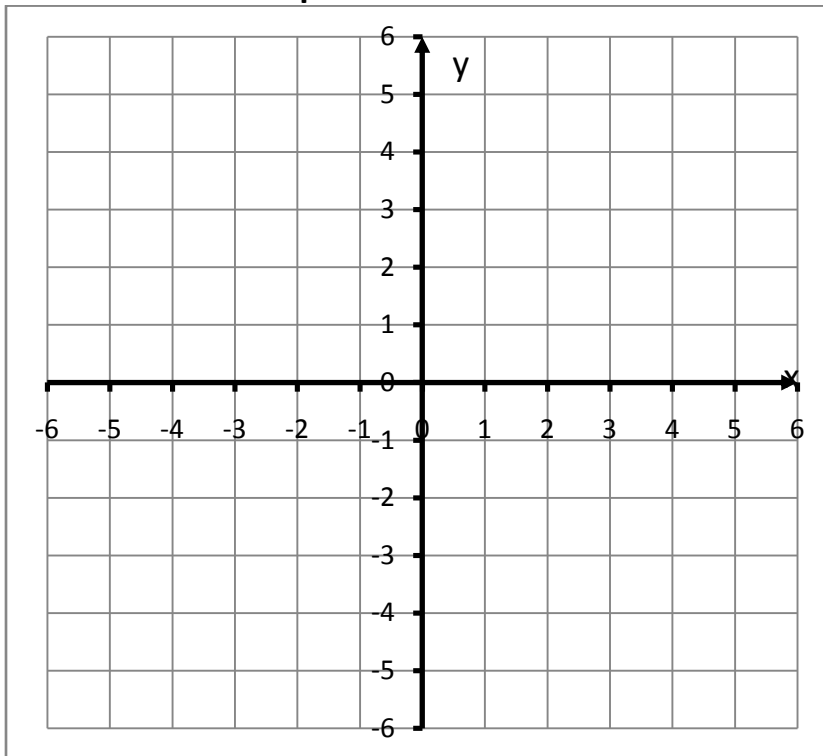
Slope-intercept form

$$y = mx + b$$

Term	Symbol	Meaning
Slope	m	$\frac{\text{change in } y}{\text{change in } x}$ or $\frac{\text{rise}}{\text{run}}$
Y-intercept	b	Value of y when $x = 0$

What are the slope and y-intercept for $y = 2x - 1$?

More about the slope



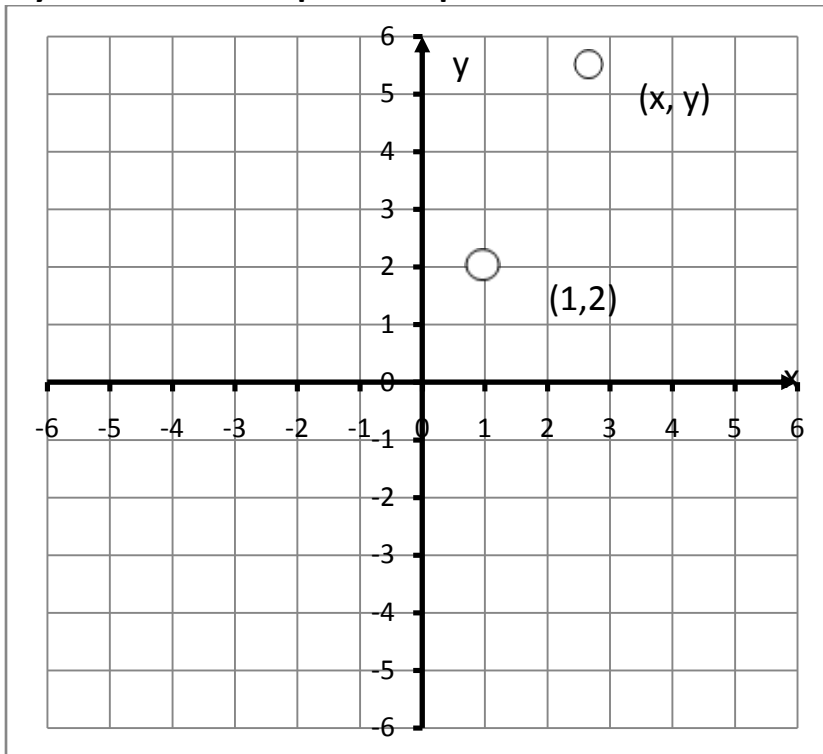
What does it mean if $m < 0$?

If $m = 0$?

What is the slope of a vertical line?

FINDING AN EQUATION OF A LINE:

If you know the slope and 1 point on the line: use the definition of slope



Suppose a line has slope 2 and contains the point (1,2)

1. Find another point on the line and write its coordinates
2. Write an equation for the line
Solution: remember $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$
3. The equation above is in *point-slope* form. Convert it to *slope-intercept* form
4. Find the x-intercept of the line.
5. Sketch the line

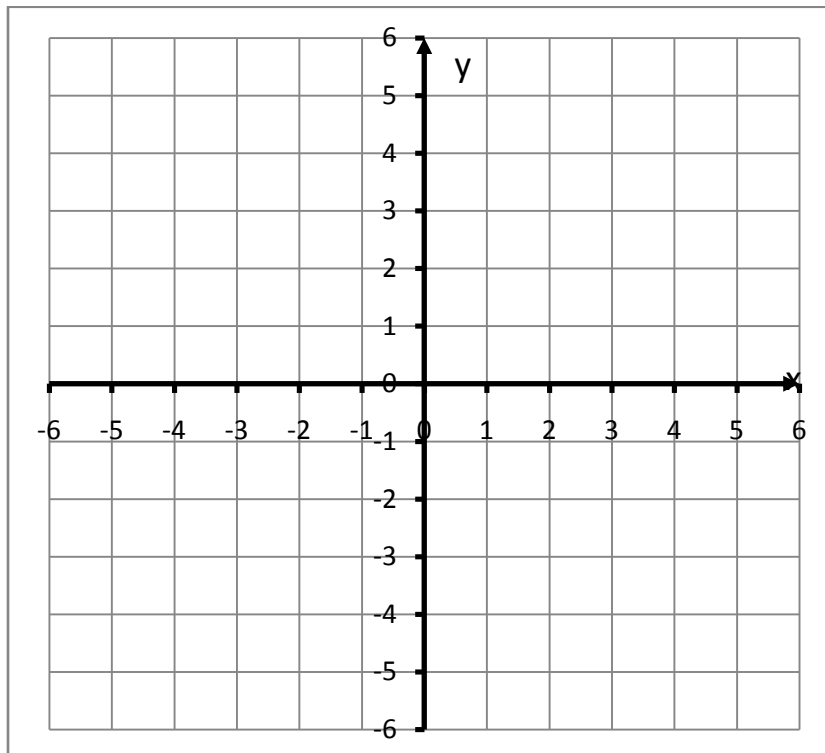
Exercise: A line has slope -1 and contains the point $(-1,4)$.

1. Write an equation for the line

2. Put the line in slope-intercept form

3. Find the x-intercept

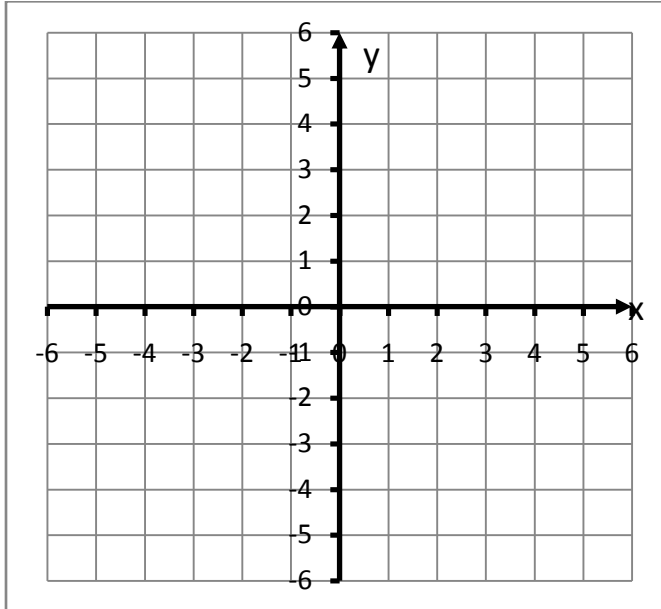
4. Sketch the line



If you know 2 points on the line: calculate the slope and then use the point-slope form

Suppose the points (3,2) and (-2,4) are on a line.

1. Plot and label the points:



2. $slope = \frac{\text{change in } y}{\text{change in } x} = \frac{\quad}{\quad} = \quad =$

3. Use this slope and the point (-2,4) to find an equation of the line

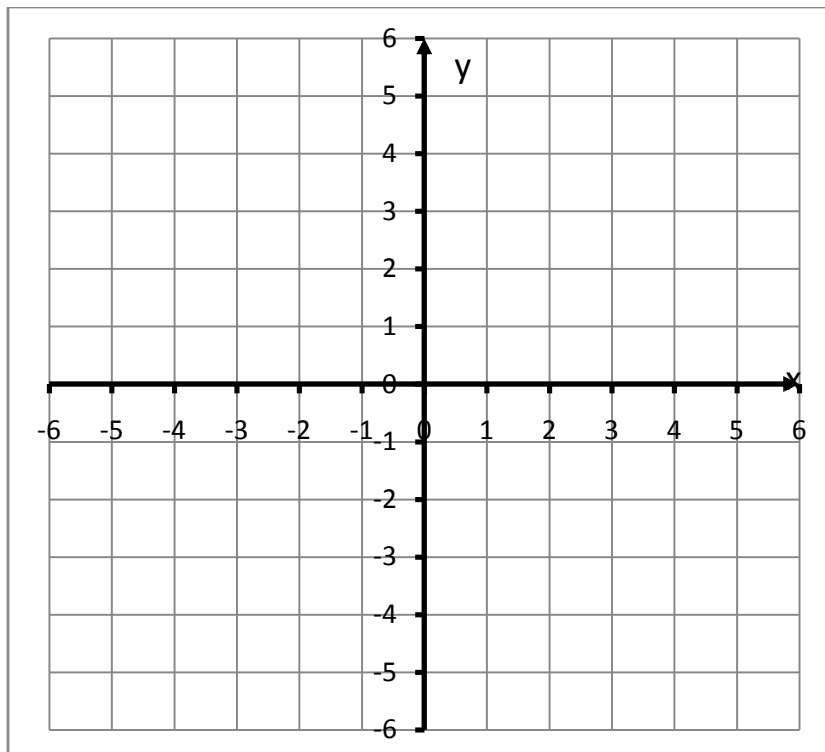
4. Put the equation in slope-intercept form

5. Find the x-intercept

6. Sketch the line

Exercise: a line contains the points $(-2,-4)$ and $(1,2)$.

1. Write an equation of the line
2. Put the equation in slope-intercept form
3. Sketch the line and label both x- and y-intercepts.

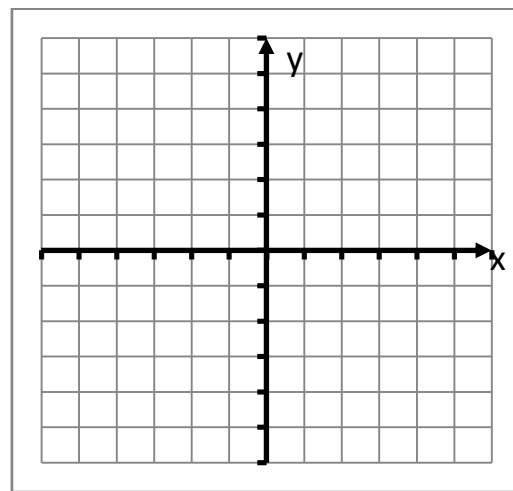
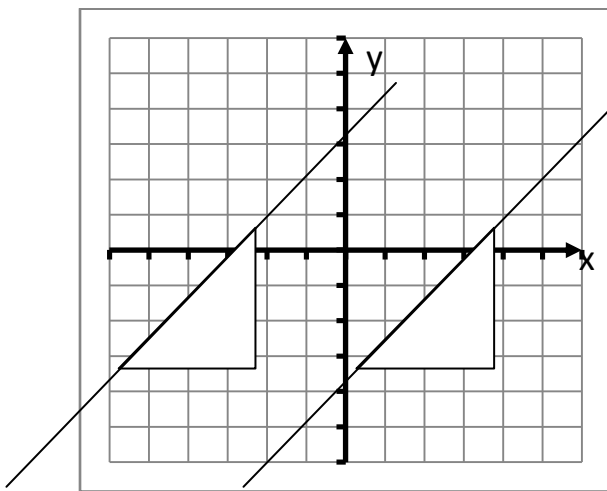


Slopes of parallel and perpendicular lines:

Let two lines have slopes m_1 and m_2 .

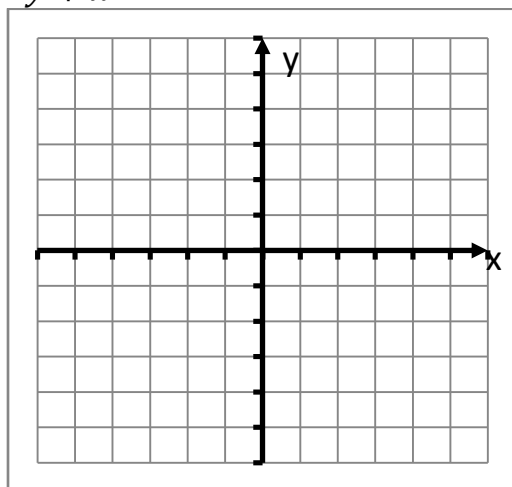
The lines are <i>parallel</i> , if and only if:	$m_1 = m_2$
Assuming neither slope is zero, the lines are <i>perpendicular</i> , if and only if:	$m_1 m_2 = -1$ Which means $m_1 = -\frac{1}{m_2}$ and $m_2 = -\frac{1}{m_1}$

Why parallel lines have the same slope, and vice versa: the two triangles below are *similar*.

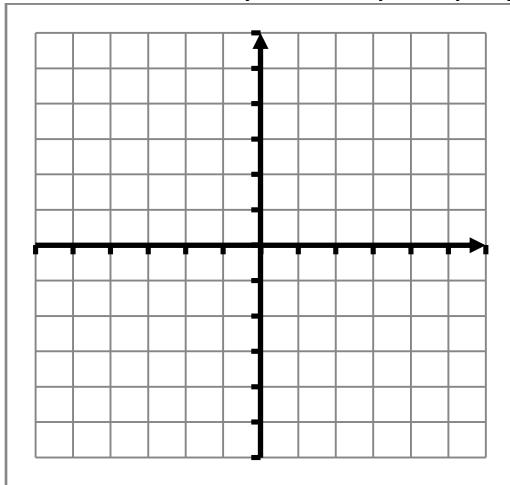


What kind of line is perpendicular to a line with slope=0?

Example: find a line that contains the point (1,-2) and is parallel to the line $2y + x = 2$



Find a line with y-intercept 3, perpendicular to the line $y = 3x + 4$



Applications

General approach to word problems

- What quantities are involved? Define variables and units.
- What are you being asked?
- What information are you given about the quantities? Write that information in terms of the variables (write equations)
- Solve the equation(s)
- Write the answer in words, and remember to include the units.

Electricity Rates in Florida Florida Power & Light Company supplies electricity to residential customers for a monthly customer charge of \$5.25 plus 6.787 cents per kilowatt-hour for up to 750 kilowatt-hours.

- (a) Write an equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month, $0 \leq x \leq 750$.
- (b) Graph this equation.
- (c) What is the monthly charge for using 200 kilowatt-hours?
- (d) What is the monthly charge for using 500 kilowatt-hours?
- (e) Interpret the slope of the line.

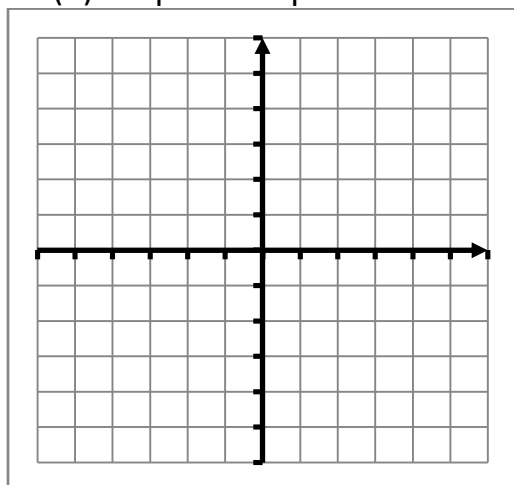
QUANTITIES

UNITS

SYMBOL

- (a) This might be a linear equation. If it is, what items do you need to know to write a linear equation? Then find the values of those items

(b) Graph the equation



(c)

(d)

(e)

QUADRATIC EQUATIONS

are equations of the form $ax^2 + bx + c = 0$ - or that can be put in that form.

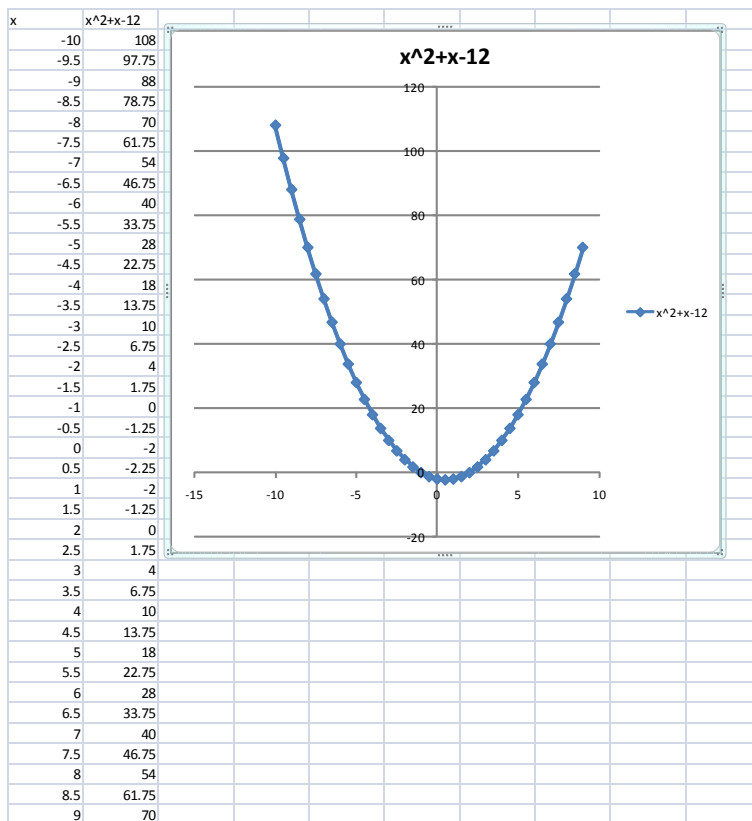
Graphical solution:

Consider $x^2 - x - 2 = 0$

Graph $y = x^2 - x - 2$

TI83 or TI84:

Excel:



Solve by factoring:

$$x^2 - x - 2 = 0$$

$$x^2 - x - 2 = (x + a)(x + b)$$

$$ab = -2 \quad \text{guess: } (x \quad)(x \quad) \text{ or } (x \quad)(x \quad)$$

$$ax + bx = -x$$

$$x^2 - x - 2 = (x \quad)(x \quad)$$

So

$$x^2 - x - 2 = 0$$

$$(x \quad)(x \quad) = 0$$

Solutions:

Exercises:

Solve $x^2 - 9 = 0$ by factoring

Now try $x^2 - 2x - 2 = 0$

Factoring is intelligent guessing, efficient for finding INTEGER solutions.

Solve by finding a square root:

Solve $x^2 = 4$

$x =$

Solve $(x - 1)^2 = 3$

$x - 1 =$

$x =$

Completing the square: example

$$x^2 - 2x - 2 = 0$$

Basic idea: turn this into $(x \quad)^2 = \underline{\quad}$

1. Put the constant alone on the right side

$$x^2 - 2x - 2 = 0$$

$$x^2 - 2x = 2$$

2. Make a square on the left side.

How can you get $-2x$?:

$$x^2 - 2x \rightarrow (x \quad)^2$$

3. "complete the square:"

$$(x \quad)^2 = x^2 - 2x \underline{\quad}$$

$$(x \quad)^2 \underline{\quad} = x^2 - 2x$$

$$x^2 - 2x = (x \quad)^2 \underline{\quad}$$

4. Substitute back into $x^2 - 2x = 2$

$$x^2 - 2x = (x \quad)^2 \underline{\quad} = 2$$

$$(x \quad)^2 = 2 \underline{\quad}$$

Exercises: complete the square for each of the following

1. $x^2 + 6x$

2. $x^2 - 3x$

Solve each of the following

3. $x^2 + 2x - 2 = 0$

4. $2x^2 - 10x + 4 = 0$ Hint – start by dividing both sides by 2

5. How would you start solving $-x^2 + 4x + 1 = 0$?

6. Solve $x^2 + 4 = 0$

This has no *real* solutions, (but does have solutions involving the number i
...we will discuss next class)

Solve using the quadratic formula

If $ax^2 + bx + c = 0$ then completing the square leads to the *quadratic formula* for the solutions: (see p. 117 of the textbook for details)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term $b^2 - 4ac$ is called the *discriminant* and shows how many real solutions there are

If $b^2 - 4ac$ is:	# real solutions:	Example: Calculate discriminant	Graph on your calculator and sketch here
> 0	2	$x^2 + 4x = 0$	$y = x^2 + 4x$
$= 0$	1, of <i>multiplicity=2</i>	$x^2 + 4x + 4 = 0$	$y = x^2 + 4x + 4$
< 0	none	$x^2 + 4x + 6 = 0$	$y = x^2 + 4x + 6$

Exercises:

How many real solutions do each of the following equations have?

1. $x^2 + 4x + 7 = 0$

2. $25x^2 - 20x + 4 = 0$

3. $2x^2 - 3x - 7 = 0$

Find the real solutions, if any, using the quadratic formula:

4. $6x^2 + 7x - 20 = 0$

5. $x^2 + x = 1$