Standard Form: $(x-h)^2+(y-k)^2=r^2$

- (x,y) is a point on the circle
- Center of the circle is (h,k)
- Radius of the circle is r.
- Intercepts can conveniently be calculated with this form.

General form: $x^2+y^2+ax+by+c=0$

- Not usually the most useful form for analyzing the circle – but you may see equations of circles in this or other forms that are not the standard form.
- To find center and radius, convert to standard form.

Examples

1. $(x-3)^2+(y-5)^2=16$

Center: (3,5)

Radius: $r=\sqrt{16}=4$

2. $(x+2)^2+(y-5)^2=36$
   $(x-(-2))^2+(y-5)^2=36$

Center (-2,5)

Radius $r=\sqrt{36}=6$

3. $x^2+(y-2)^2=4$
   $(x-0)^2+(y-2)^2=4$

Center (0,2)

Radius $r=\sqrt{4}=2$

4. $(x-1)^2+(y+2)^2=7$
   $(x-1)^2+(y-(-2))^2=7$

Center (1,-2)

Radius $r=\sqrt{7}$

5. $2(x-3)^2 + 2(y+1)^2 = 10$
$\frac{2(x-3)^2}{2} + \frac{2(y+1)^2}{2} = \frac{10}{2}$
$(x-3)^2 + (y+1)^2 = 5$
$(x-3)^2 + (y-(-1))^2 = 5$

Center (3,-1)

Radius $r = \sqrt{5}$

6. $2(x+1)^2 + (y-1)^2 = 10$

NOT a circle – cannot be put in the form $(x-h)^2 + (y-k)^2 = r^2$

7. $(x-7)^2 - (y+1)^2 = 10$

NOT a circle – due to minus sign.

8. $x^2 + y^2 + 2x + 4y - 11 = 0$

Strategy: complete the square for x and complete the square for y.

$x^2 + 2x + (\,) + y^2 + 4y + (\,) - 11 = 0 + (\,) + (\,)$

$x^2 + 2x + 1 + y^2 + 4y + 4 - 11 = 0 + 1 + 4$

$(x+1)^2 + (y+2)^2 - 11 = 5$

$(x+1)^2 + (y+2)^2 = 16$

$(x-(-1))^2 + (y-(-2))^2 = 16$

Center (-1,-2)

Radius $r = \sqrt{16} = 4$

9. $x^2 + y^2 + x - 6y - 2 = 0$

Strategy: complete the square for x and complete the square for y.

$x^2 + x + (\,) + y^2 - 6y + (\,) - 2 = 0 + (\,) + (\,)$

$x^2 + x + \frac{1}{4} + y^2 - 6y + 9 - 2 = 0 + \frac{1}{4} + 9$

$(x + \frac{1}{2})^2 + (y - 3)^2 - 2 = 9\frac{1}{4}$

$(x + \frac{1}{2})^2 + (y - 3)^2 = 11\frac{1}{4}$

$(x - (-\frac{1}{2}))^2 + (y - 3)^2 = 11\frac{1}{4} = \frac{45}{4}$

Center (-1/2, 3)

Radius $r = \sqrt{\frac{45}{4}} = \frac{\sqrt{45}}{2}$

10. $x^2 + y^2 + 6x + 2y - 1 = 0$

Strategy: complete the square for x and complete the square for y.

$x^2 + 6x + (\,) + y^2 + 2y + (\,) + 12 = 0 + (\,) + (\,)$

$x^2 + 6x + 9 + y^2 + 2y + 1 + 12 = 0 + 9 + 1$

$(x + 3)^2 + (y + 1)^2 + 12 = 10$

$(x + 3)^2 + (y + 1)^2 = -2$

NOT the equation of a circle . $r = \sqrt{-2}$ is not a real number.

11. Looking at Example 1 again – and this time find the intercepts

$(x-3)^2+(y-5)^2=16$

Center: (3,5)

Radius: $r=\sqrt{16}=4$

Y intercepts: set x=0

$(0-3)^2+(y-5)^2=16$

$9+(y-5)^2=16$

$(y-5)^2=16-9=7$

$(y-5)=\pm\sqrt{7}$

$y=5\pm\sqrt{7}$

$\left(0,5-\sqrt{7}\right)$ and $\left(0,5+\sqrt{7}\right)$

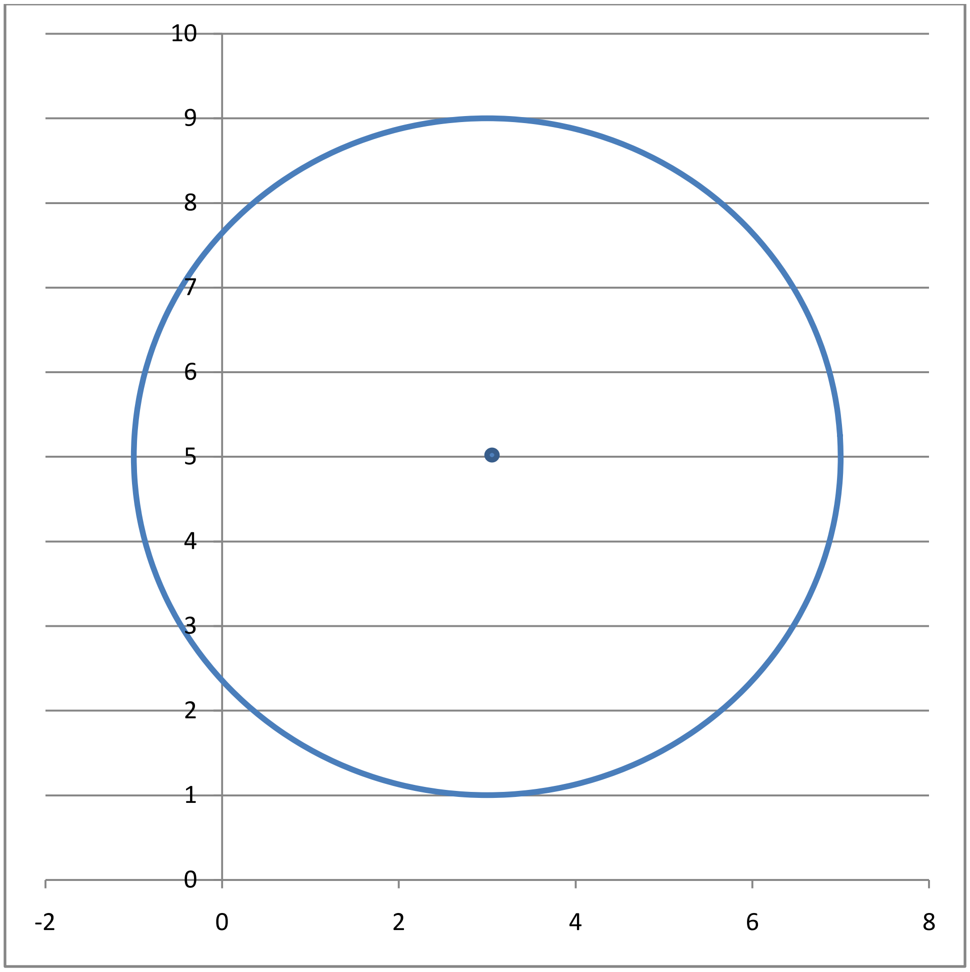
X intercepts: set y=0

$(x-3)^2+(0-5)^2=16$

$(x-3)^2+25=16$

$(x-3)^2=16-25=-9$

There are no X-intercepts. Look again at the center (3,5) and radius=4. Lowest possible y value is 5-4=1 so the circle never crosses the x-axis. ***SEE ILLUSTRATION ON NEXT PAGE***


10
9
8
7
6
5
4
3
2
1
0
-2
0
2
4
6
8