## Math 120 Notes 6.8 – Phase Shift and Sinusoidal Regression

**I.** Phase shift.

**A.** Graph the following in addition to $y = \sin(x)$ on the calculator. Identify any patterns.

**1.** $y = \sin\left(x + \frac{\pi}{4}\right)$ **2.** $y = \sin\left(x - \frac{\pi}{4}\right)$ **3.** $y = \sin\left(x + \frac{\pi}{2}\right)$

**B.** How do we move the sine function left and right?

$y = A\sin(Bx + C) + D$

amp = $|A|$ reflect if $A < 0$

per = $\frac{2\pi}{B}$ phase shift (where the graph starts) = $-\frac{C}{B}$

midline = D

**C.** Find the function that transforms the function $y = \cos(x)$ by

**1.** Shifting 40° to the right.

**2.** Shifting $\pi$ to the left, and having an amplitude of 4.

**3.** Shifting $\pi/6$ to the right, having an amplitude of ¾, and reflecting across the *x*-axis.

① $y = \cos(x - 40°)$

② $y = 4\cos(x + \pi)$

③ $y = -3/4\cos(x - \pi/6)$

**II.** Phase shift and period

**A.** If the function is in the form $y = \sin(\alpha x + \beta)$, how do we find the period and phase shift?

**B.** For each of the following functions, identify the [a] amplitude, [b] period, [c] phase shift, [d] vertical shift, and [e] reflection.

**1.** $y = 9\sin(2x + 4\pi) - 4$

B = 2, C = 4π

amp = 9

per = $\frac{2\pi}{2} = \pi$

phase shift = $-\frac{4\pi}{2} = -2\pi$

midline = −4

reflect? no

**2.** $y = -4\cos\left(3x - \frac{3\pi}{2}\right) + 1$

amp = 4 ; reflect? yes

per = $\frac{2\pi}{3}$

phase shift = $+\frac{3\pi}{2} \div 3 = \frac{3\pi}{2} \cdot \frac{1}{3} = \frac{\pi}{2}$

midline = 1

| | |
|---|---|
| $\cos(-x) = \cos(x)$ | $\sin(-x) = -\sin(x)$ |

**3.** $y = -3\sin\left(2x - \frac{\pi}{2}\right) + 5$

amp = 3 ; reflect? yes

per = $\frac{2\pi}{2} = \pi$

phase shift = $+\frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

midline = 5

**4.** $y = 7\cos\left(-3x - \frac{5\pi}{2}\right) - \pi$

$y = 7\cos\left(-1\left(3x + \frac{5\pi}{2}\right)\right) - \pi$

$= 7\cos\left(3x + \frac{5\pi}{2}\right) - \pi$

amp = 7 ; reflect? no

per = $\frac{2\pi}{3}$

phase shift = $-\frac{5\pi}{2} \div 3 = -\frac{5\pi}{2} \cdot \frac{1}{3} = -\frac{5\pi}{6}$

mid = −π

**C.** Use the trig graphs below to graph the following equations

**1.** $y = -3\sin(-2x + \pi) - 1$

$y = -3\sin(-1(2x - \pi)) - 1$

$y = 3\sin(2x - \pi) - 1$

amp = 3 ; reflect? no

per = $\frac{2\pi}{2} = \pi$ (key pts = $\frac{\pi}{4}$)

phase shift = $+\frac{\pi}{2}$ ← start here

mid = −1

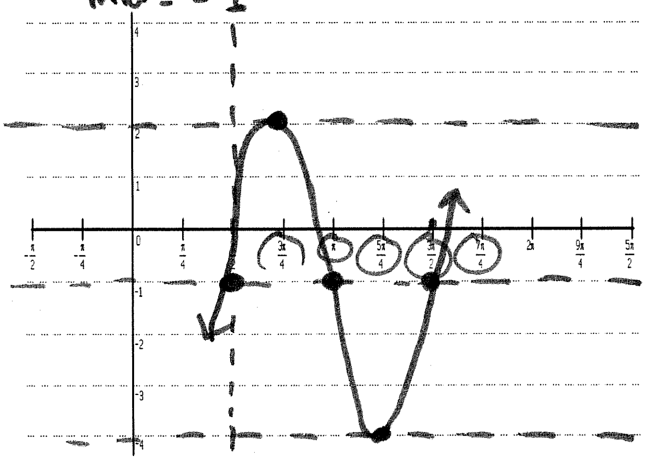
**2.** $y = -2\cos\left(2x - \frac{\pi}{2}\right) + 2$

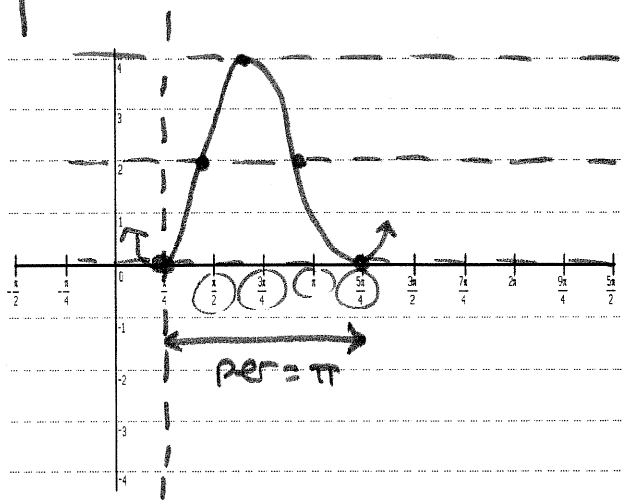
amp = 2 ; reflect? yes

per = $\frac{2\pi}{2} = \pi$ key pts = $\pi/4$

phase shift = $+\frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ start

mid = 2





**Math 120 Notes 7.1 – Inverse Trig functions**

**I.** Review of Inverse Functions

**A.** Fries example 50¢/box

$f^{-1}(x)$

| x | f(x) |
|---|---|
| 0 | 0 |
| 1 | .50 |
| 2 | 1.00 |
| 3 | 1.50 |
| 4 | 2 |

Coupon 4 boxes for $1

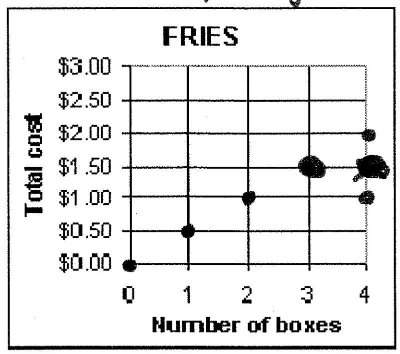
*sale*

4 boxes for $1.50

one-to-one

each x gives only one unique y-value.

**B.** Inverse: If f(x) is the cost for *x* boxes of fries, interpret the meaning of




**1.** $f(10) = 5$

10 boxes costs $5

**2.** $f^{-1}(5) = 10$

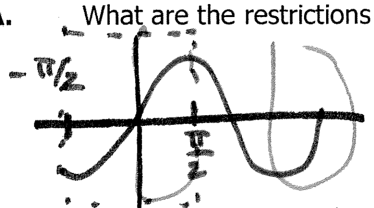
$5 gets you 10 boxes

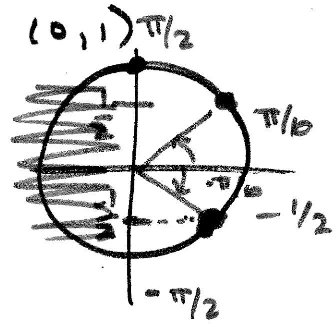
HLT

**II.** Inverse sine/cosine

**A.** What are the restrictions on the domain of $y = \sin^{-1}(x)$?

from $-\pi/2$ to $\pi/2$



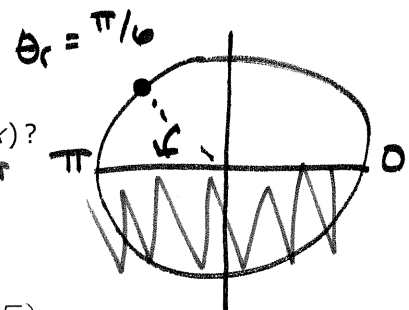




1. Evaluate $\sin^{-1}(1)$, $\sin^{-1}\left(\frac{1}{2}\right)$, and $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ without using the calculator

$\sin^{-1}(1) = \pi/2$ because $\sin(\pi/2) = 1$

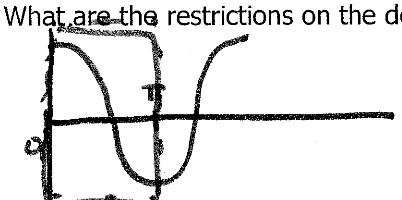
$\sin^{-1}(1/2) = \pi/6$ ; $\sin^{-1}(-\sqrt{3}/2) = -\pi/3$

2. Evaluate $\sin^{-1}(0.8)$ and $\sin^{-1}\left(-\frac{3}{4}\right)$ using the calculator, round to three decimals.




**B.** What are the restrictions on the domain of $y = \cos^{-1}(x)$?

from 0 to $\pi$




1. Evaluate $\cos^{-1}(-1)$, $\cos^{-1}\left(\frac{1}{2}\right)$, and $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ without using the calculator

$\cos^{-1}(-1) = \pi$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

$\cos^{-1}(1/2) = \pi/3$

2. Evaluate $\cos^{-1}(0.75)$ and $\cos^{-1}\left(-\frac{4}{7}\right)$ using the calculator, round to three decimals.

**C.** Composite functions

1. $\sin^{-1}\left[\sin\left(\frac{\pi}{6}\right)\right]$

2. $\cos\left[\cos^{-1}\left(\frac{1}{8}\right)\right]$